CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY

CEBI WORKING PAPER SERIES

Working Paper 09/24

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ISSN 2596-447X

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Household Bargaining with Limited Commitment: A Practitioner's Guide*

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May 15, 2024

Abstract

In this guide, we introduce the limited commitment model of dynamic household bargaining behavior over the life cycle. The guide is intended to make the limited commitment model more accessible to researchers who are interested in studying intra-household allocations and divorce over the life cycle. We mitigate computational challenges by providing a flexible base of code that can be customized and extended to the specific use case. The main contribution is to discuss practical implementation details of the model class, and provide guidance on how to efficiently solve limited commitment models using state-of-the-art numerical methods. The setup and solution algorithm is presented through a stylized example of dynamic consumption allocation and includes accompanying Python and C++ code used to generate all results.

JEL-codes: D13, D15, C61, C63, C78.

Keywords: Household Bargaining, limited commitment, life cycle, couples, numerical dynamic programming.

*All errors are ours. Comments and suggestions are welcome! The Center for Economic Behavior and Inequality (CEBI) at the University of Copenhagen is financed by grant DNRF134 from the Danish National Research Foundation. we acknowledge financial support from the Carlsberg Foundation, grant CF22-0317.

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1 Introduction

Household behavior is the result of individuals' joint decisions - but these individuals may not have perfectly aligned interests. To study household dynamics, it is therefore of key importance to take the intra-household decision problem seriously. While most existing economic research has ignored dynamic bargaining and bargaining altogether, a growing literature applies the so-called "limited commitment" model. While this framework provides a rich description of dynamic bargaining of couples and allows for endogenous divorce, it is computationally complex to implement and time consuming to numerically solve. The limited commitment model is based on the theoretical work of Marcet and Marimon (1992, 2019); Ligon (2002) and Ligon, Thomas and Worrall (2002) and popularized by Mazzocco (2007). One of the earliest implementations is in Attanasio and Ríos-Rull (2000). See also Voena (2015); Bronson (2015); Mazzocco, Ruiz and Yamaguchi (2013); Bronson and Mazzocco (2021); Low, Meghir, Pistaferri and Voena (2018) for empirical implementations of the limited commitment framework. While the application of the modelling framework is growing rapidly, the implementation details can be hard to get from published papers. We fill this gap.

We provide a unified notation that enables us to clearly describe the key features of the limited commitment dynamic bargaining model. We also describe how to numerically solve the model using numerical dynamic programming tools, providing two implementation tricks to speed up the solution substantially. First, we give a novel way to improve speed and accuracy of the solution by refined calculation of individual indifference points. Second, we show how to implement the fast and accurate endogenous grid method (EGM) proposed by Carroll (2006), combining recent work by Druedahl and Jørgensen (2017) and Hallengreen, Jørgensen and Olesen (2024).

We discuss the framework and numerical solution in general and through a concrete example of dynamic consumption allocation. We provide Python and C++ code used to generate all results.[1]

In the next Section we propose a general notation and describe the limited commitment framework. In Section 3 we describe how models within this framework can be solved. In Section 4 we describe the formulatin of the model, the numerical solution and simulations of a concrete example. In Section 5 we conclude with a discussion.

[1] All code is available from the accompanying GitHub repository, https://github.com/ThomasHJorgensen/HouseholdBargainingGuide.

2 The Limited Commitment Framework

The limited commitment framework takes seriously the fact that utility is not perfectly transferable between household members and that each member cannot perfectly commit to future allocations. The theoretical foundations are due to Marcet and Marimon (1992, 2019) and Ligon, Thomas and Worrall (2002). While these authors focus on other situations with bargaining, recent literature has used these ideas in the context of household decisions.

Attanasio and Ríos-Rull (2000) was one of the first empirical papers to implement the limited commitment model. They study how the intra-household insurance mechanism is distorted by the lack of full commitment. More recently, following the work of Mazzocco (2007), the limited commitment model has been adopted by several studies cited in the Introduction. The limited commitment framework has recently been used to study, e.g., the labor supply of couples (Mazzocco, Ruiz and Yamaguchi, 2013); the effect of divorce laws on savings and labor supply (Voena, 2015); how the social safety net affects individual well-being (Low, Meghir, Pistaferri and Voena, 2018); and how joint taxation of couples affects their labor supply (Bronson and Mazzocco, 2021). Chiappori and Mazzocco (2017) provides an excellent overview of the limited commitment model and the relation to other frameworks, such as the unitary model.

The main idea in the limited commitment framework is that in each period, both household members evaluate whether it is preferable for them to remain in the couple or if it is more valuable to divorce. Throughout we will use divorce and singlehood interchangeable and also with partnership and marriage. Other outside options than divorce, such as non-cooperation, could also be used (see e.g. Lundberg and Pollak, 1993). The value of being in the couple depends on the bargaining power of each member. Only if the outside option is preferable for one of the members, the bargaining power is renegotiated in search for an updated allocation that can sustain the marriage. If that is not possible, the couple divorces. In the following, we will be much more explicit about the setup.

We begin by describing the recursive problem and define the outside options and related forward looking participation constraints that are the defining feature of this model that separates it from a standard dynamic programming problem. In the Supplementary Material, we motivate the following formulation through a more technical formulation based on the general setup in Marcet and Marimon (2019), applied to the household setting.

2.1 Notation and Setup

Before continuing, a comment on the current notation is in place. In what follows, we will denote the value of household member j from transitioning from x to y as $V_{j,t}^{x\rightarrow y}(\bullet)$ where $x, y \in \{m, s\}$ with m denoting "married" and s denoting "single". Similarly, we will denote $V_{j,t}^{x}(\bullet)$ as the value of member j from entering period t as $x \in \{m, s\}$. we will not distinguish between cohabitation and marriage here.

We collect state variables, such as savings, human capital, etc. in the vector $\mathcal{S}_t$. Note that this set contains all relevant state variables in excess of marital status (which we will indicate with function superscripts) and the state measuring the relative bargaining power of household member 1, μ_{t-1}. This co-state is a key element of the limited commitment model and will be of great focus below. Given the set of state variables, couples choose the vector $\mathcal{C}_t$. We assume that the states transition following some known distribution,

$$\mathcal{S}_{t+1} \sim \Gamma(\mathcal{S}_t, \mathcal{C}_t)$$

with $\mathcal{S}_0$ given, and that choices potentially have to satisfy some additional constraints, such as a budget constraint. The important complication in the limited commitment framework is the presence of forward-looking participation constraints, and we will thus ignore all other constraints in the following exposition. In the example below, we will include all the constraints in the formulation to be precise.

Finally, we denote the beginning-of-period bargaining weight coming into period t as μ_{t-1}. We do this to acknowledge that the bargaining weight in period t is *endogenously updated* if the participation constraints are binding. Others, such as e.g. Voena (2015) and Bronson (2015) seem to denote the beginning-of-period bargaining weight as μ_t. Attanasio and Ríos-Rull (2000) seem to be assuming that the bargaining weight is not updated in the current period but only from the next period onward. To reduce confusion, we have chosen to emphasize the timing of events by denoting the beginning-of-period bargaining weight as μ_{t-1}. See also the Supplemental Material for why we choose this notation. As we will see, the co-state will transition following

$$\mu_t = \mu_t^{\star}(\mathcal{S}_t, \mu_{t-1})$$

where this updating rule is at the heart of the limited commitment model and will be derived below.

2.2 A Recursive Formulation

The following formulation is inspired by the expositions in e.g. Voena (2015) and Bronson (2015). See also Ligon (2002) and Ligon, Thomas and Worrall (2002). Our main contribution here, besides clearly stating all model objects using the notation above, is to provide a compact mathematical formulation of the bargaining process and the evolution of the endogenous bargaining power.

The value of *entering* a period as married given the beginning of period bargaining weight, μ_{t-1}, is

$$V_{j,t}^{m}(\mathcal{S}_t, \mu_{t-1}) = D_t^{\star} V_{j,t}^{m\to s}(\mathcal{S}_{j,t}) + (1 - D_t^{\star}) V_{j,t}^{m\to m}(\mathcal{S}_t, \mu_t^{\star})$$

for $j \in \{1,2\}$, where $D_t^{\star} = 1$ denotes the endogenous choice of divorce, and $\mu_t^{\star}$ denotes the outcome of bargaining. We will return to how this is determined below.

The value of *transitioning* from marriage to singlehood is

$$V_{j,t}^{m\to s}(\mathcal{S}_{j,t}) = \max_{\mathcal{C}_{j,t}} U_j(\mathcal{C}_{j,t}, \mathcal{S}_{j,t}) + \beta \mathbb{E}_t[V_{j,t+1}^{s}(\mathcal{S}_{j,t+1}, \mathcal{S}_{j,t+1}^{p})] \quad (1)$$

for $j \in \{1,2\}$ where $\mathcal{S}_{j,t} \subseteq \mathcal{S}_t$ and the states of a potential partner is $\mathcal{S}_t^{p} \subseteq \mathcal{S}_{j,t}$. The value of the outside option is a key element in the subsequent bargaining process of couples and ultimately determines if a couple remains together or divorces. It also plays a key role in determining the bargaining weight, $\mu_t^{\star}$. The outside option could be characterized by other regimes than singlehood, but this setup is the most common assumption in the literature. See e.g. Lundberg and Pollak (1993) for an alternative in which the outside option (threat-point) is a noncooperative equilibrium within marriage. A cost of divorce could be subtracted in eq. (1) as in e.g. Bronson (2015).

The expected value of *entering* next period as single is for $j \in \{1,2\}$

$$\begin{aligned}\mathbb{E}_t[V_{j,t+1}^{s}(\mathcal{S}_{j,t+1}, \mathcal{S}_{j,t+1}^{p})] = \int \Big[& M_{j,t+1}^{\star} V_{j,t+1}^{s\to m}(\mathcal{S}_{j,t+1}, \mathcal{S}_{j,t+1}^{p}) \\ & + (1 - M_{j,t+1}^{\star}) V_{j,t+1}^{s\to s}(\mathcal{S}_{j,t+1}) \Big] \Gamma(d\mathcal{S}_{j,t+1}, d\mathcal{S}_{j,t+1}^{p})\end{aligned}$$

where $\Gamma(\mathcal{S}_{j,t+1}, \mathcal{S}_{t+1}^{p})$ is the joint pdf over own states and those of a potential partner. $M_{j,t}^{\star} = 1$ denotes the situation in which a single individual marries. This could be a random variable or an outcome of active choice. We will focus primarily on couples and will thus let this process be unspecified in the general formulation here. Likewise, we will not discuss $V^{s\to m}_{j,t}(\mathcal{S}_{j,t}, \mathcal{S}^p_t)$ and $V^{s\to s}_{j,t}(\mathcal{S}_{j,t})$ as these generally depend on the assumed repartnering process for singles. For example, if singlehood is an absorbing state, then $M^\star_{j,t} = 0$ and $V^s_{j,t}(\mathcal{S}_{j,t}) = V^{s\to s}_{j,t}(\mathcal{S}_{j,t})$ for all j and t. In our concrete example below, we will discuss these objects in more detail.

The value of *remaining* a couple is determined in two steps. In the first step, the value of marriage at the current level of bargaining power, μ_{t-1}, is calculated. In the second step, bargaining might take place. In what follows, all objects are defined for some arbitrary value of μ (not only μ_{t-1}) since that will be helpful in explaining the bargaining process.

Let the *value of choice* associated with *i)* remaining married and *ii)* choosing $\mathcal{C}_t$, given some bargaining weight μ, be

$$v_t(\mathcal{S}_t, \mathcal{C}_t, \mu) = \mu v_{1,t}(\mathcal{S}_t, \mathcal{C}_t, \mu) + (1-\mu) v_{2,t}(\mathcal{S}_t, \mathcal{C}_t, \mu) \tag{2}$$

where for $j \in \{1, 2\}$

$$v_{j,t}(\mathcal{S}_t, \mathcal{C}_t, \mu) = U_j(\mathcal{C}_t, \mathcal{S}_t) + \beta \mathbb{E}_t[V^m_{j,t+1}(\mathcal{S}_{t+1}, \mu)]$$

is individual j's value from remaining married at $\mathcal{C}_t$ and μ. For this reason, the next period value function is $V^m_{j,t+1}(\mathcal{S}_{t+1}, \mu)$ since member j will enter period $t+1$ as married by assumption. Then

$$\tilde{\mathcal{C}}_t(\mathcal{S}_t, \mu) = \arg\max_{\mathcal{C}_t} v_t(\mathcal{S}_t, \mathcal{C}_t, \mu) \tag{3}$$

is the solution for a couple conditional on them remaining married with bargaining weight μ, and state variables $\mathcal{S}_t$.We denote the solution with a tilde, $\tilde{\mathcal{C}}_t(\mathcal{S}_t, \mu)$, and not a $\star$ to signify that the final optimal choices of a couple with states $(\mathcal{S}_t, \mu_{t-1})$ might be different from $\tilde{\mathcal{C}}_t(\mathcal{S}_t, \mu_{t-1})$ due to bargaining, as will become clear very soon. Note that inserting the optimal choices into the individual value of choice function gives the value associated with remaining married,

$$\tilde{V}^{m\to m}_{j,t}(\mathcal{S}_t, \mu) = v_{j,t}(\mathcal{S}_t, \tilde{\mathcal{C}}_t(\mathcal{S}_t, \mu), \mu)$$

for this value of μ. Inserting the beginning-of-period bargaining power, and allowing for the bargaining process below gives the actual value of remaining a couple

$$V_{j,t}^{m\rightarrow m}(\mathcal{S}_t,\mu_{t-1}) = v_{j,t}(\mathcal{S}_t,\tilde{\mathcal{C}}_t(\mathcal{S}_t,\mu_t^\star),\mu_t^\star)$$

with that set of states. Below, we now describe how the bargaining process leads to $\mu_t^\star$.

The next step is to check whether the *individual participation constraints* are satisfied. For this purpose, denote the *marital surplus* of member j given states and a bargaining power of μ as

$$S_{j,t}(\mathcal{S}_t,\mu) = \tilde{V}_{j,t}^{m\rightarrow m}(\mathcal{S}_t,\mu) - V_{j,t}^{m\rightarrow s}(\mathcal{S}_t). \tag{4}$$

The forward-looking participation constraints are satisfied in the current states if $S_{j,t}(\mathcal{S}_t,\mu_{t-1}) \geq 0$ for $j \in \{1,2\}$. Note that μ_{t-1} is inserted here. There are now three relevant situations depending on the marital surplus:[2]

1. If both spouses have a positive marital surplus with the beginning-of-period bargaining weight, μ_{t-1}, i.e. $S_{j,t}(\mathcal{S}_t,\mu_{t-1}) \geq 0$ for $j \in \{1,2\}$, they remain married and $D_t^\star = 0$. Furthermore, the bargaining weight is not updated and $\mu_t = \mu_t^\star = \mu_{t-1}$. Thus, the optimal choices are $\mathcal{C}_t^\star(\mathcal{S}_t,\mu_{t-1}) = \tilde{\mathcal{C}}_t(\mathcal{S}_t,\mu_{t-1})$ and the value of starting as married and remaining married is identical, $V_{j,t}^{m}(\mathcal{S}_t,\mu_{t-1}) = V_{j,t}^{m\rightarrow m}(\mathcal{S}_t,\mu_{t-1}) = v_{j,t}(\mathcal{S}_t,\mu_{t-1},\tilde{\mathcal{C}}_t(\mathcal{S}_t,\mu_{t-1}))$.

2. If none of the household members have positive marital surpluses, $S_{j,t}(\mathcal{S}_t,\mu_{t-1}) < 0$ for $j \in \{1,2\}$, the couple divorces and $D_t^\star = 1$. The value of entering the period as married is then $V_{j,t}^{m}(\mathcal{S}_t,\mu_{t-1}) = V_{j,t}^{m\rightarrow s}(\mathcal{S}_t)$ along with the associated optimal choices from equation (1).

3. The couple renegotiates the bargaining power if one of the spouses, say $j = 1$, has a negative marital surplus while the other spouse has a positive surplus. Let $\tilde{\mu}_1$ be the level of bargaining power that puts $S_{1,t}(\mathcal{S}_t,\tilde{\mu}_1) = 0$ such that member 1 is indifferent between remaining married and divorcing. If the other member has a positive surplus with this updated bargaining power, $S_{2,t}(\mathcal{S}_t,\tilde{\mu}_1) \geq 0$, the couple remains married, $D_t^\star = 0$. The bargaining weight is then updated to $\mu_t = \mu_t^\star = \tilde{\mu}_1$, and the value of entering and remaining married is $V_{j,t}^{m}(\mathcal{S}_t,\mu_{t-1}) = V_{j,t}^{m\rightarrow m}(\mathcal{S}_t,\mu_{t-1}) = v_{j,t}(\mathcal{S}_t,\tilde{\mu}_1,\tilde{\mathcal{C}}_t(\mathcal{S}_t,\tilde{\mu}_1))$.[3] The optimal choices are $\mathcal{C}_t^\star(\mathcal{S}_t,\mu_{t-1}) = \tilde{\mathcal{C}}_t(\mathcal{S}_t,\tilde{\mu}_1)$. If, on the other hand, member 2 prefers the outside option with this alternative allocation, $S_{2,t}(\mathcal{S}_t,\tilde{\mu}_1) < 0$, there is no feasible bargaining power allocation that can sustain

[2] See Chiappori and Mazzocco (2017, Figure 1) for a graphical illustration of the bargaining process.

[3] Ligon, Thomas and Worrall (2002) show that this updating scheme is optimal. The intuition comes from the min-max saddle-point problem in the Supplementary Material. The shadow price, $\gamma_{1,t}$, should be chosen as the *lowest* value such that the forward-looking participation constraint is satisfied. This corresponds to the lowest value of μ (since that is the weight on member 1) that puts $S_{1,t}(\mathcal{S}_t,\mu) = 0$.

marriage and the couple divorces with similar consequences as above. The process is symmetric if it is partner 2 that is unsatisfied with the initial bargaining weight.

To summarize, let $\tilde{\mu}_j$ be the level of bargaining power that puts $S_{j,t}(\mathcal{S}_t, \tilde{\mu}_j) = 0$. The bargaining power updating rule can then be formulated as

$$\mu_t = \mu_t^\star(\mathcal{S}_t, \mu_{t-1}) = \begin{cases} \mu_{t-1} & \text{if } S_{1,t}(\mathcal{S}_t, \mu_{t-1}) \geq 0 \text{ and } S_{2,t}(\mathcal{S}_t, \mu_{t-1}) \geq 0 \\ \tilde{\mu}_1 & \text{if } S_{1,t}(\mathcal{S}_t, \mu_{t-1}) < 0 \text{ and } S_{2,t}(\mathcal{S}_t, \tilde{\mu}_1) \geq 0 \\ \tilde{\mu}_2 & \text{if } S_{1,t}(\mathcal{S}_t, \tilde{\mu}_2) \geq 0 \text{ and } S_{2,t}(\mathcal{S}_t, \mu_{t-1}) < 0 \\ \emptyset & \text{else} \end{cases} \tag{5}$$

with μ_0 given, and the optimal divorce choice is

$$D_t^\star = \begin{cases} 1 & \text{if } S_{1,t}(\mathcal{S}_t, \tilde{\mu}_2) < 0 \text{ and } S_{2,t}(\mathcal{S}_t, \tilde{\mu}_1) < 0 \\ 0 & \text{else.} \end{cases} \tag{6}$$

The initial bargaining weight, μ_0, can be chosen in several ways. Some examples include cooperative Nash bargaining

$$\mu_0(\mathcal{S}_0) = \arg\max_{\mu} \left(S_{1,t}(\mathcal{S}_0, \mu)\right)^w \left(S_{2,t}(\mathcal{S}_0, \mu)\right)^{1-w}$$

with some weight w on household member 1. This could be equal weighting, $w = 0.5$. Alternatively, the marital surplus could be allocated in equal parts to each spouse,

$$\mu_0(\mathcal{S}_0) : S_{1,t}(\mathcal{S}_0, \mu) = \frac{1}{2}(S_{1,t}(\mathcal{S}_0, \mu) + S_{2,t}(\mathcal{S}_0, \mu)).$$

3 Numerical Solution

The model can be solved by standard value function iteration (VFI) following the exposition above. However, VFI can be quite slow. Here, we will discuss how models with endogenous wealth accumulation can be solved using the endogenous grid method (EGM) as proposed by Carroll (2006) using the idea in Hallengreen, Jørgensen and Olesen (2024). Since the model involves non-convex elements, such as discrete divorce choices, the extensions in Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017) can be applied. See also the nesting of EGM in VFI in Druedahl (2021). The main contribution is to show how the EGM can be applied to this framework, although marginal utility and the inverse hereof typically cannot be found in closed form, which has always been assumed in all extensions of the EGM thus far. Therefore, the approach has applicability outside the scope of the limited commitment framework. Furthermore, we propose a simple approach to evaluate the participation constraints and to update the bargaining weight, $\mu_t^\star$. Implementation details are deferred to a concrete example in the next Section.

For this exposition, we will assume that one relevant state variable is the beginning-of-period wealth, A_{t-1}, and collect the remaining state variables in $\mathcal{S}_t$. We will denote M_t as total household resources and C_t as total consumption in period t. Households thus choose consumption along with other choices, all collected in $\mathcal{C}_t$. If couples choose private and public consumption, C_t will be the sum of these components, such that

$$M_t = A_t + C_t$$

stating that all resources are consumed or saved. Furthermore, we assume that the budget constraint has the form

$$A_t = RA_{t-1} + Y_t - C_t$$

where Y_t includes income and transfers net of taxes for all household members. This means that resources follows

$$M_{t+1} = R(M_t - C_t) + Y_{t+1}.$$

We will focus on how to solve the problem of a couple under the assumption that they remain together, in (3) above. This is probably the most complicated and time-consuming part of the numerical solution algorithm. Once we have solved for $\tilde{\mathcal{C}}_t$, the algorithm is rather straightforward.

The EGM solves for total consumption in closed form. Let $U(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu)$ be the household utility of total consumption. For example, if members choose private and public consumption, the household utility function could be something like

$$\begin{aligned} U(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu) = \max_{c, c_1, c_2} & \ \mu u_1(c, c_1, \mathcal{C}_t, \mathcal{S}_t) + (1-\mu) u_2(c, c_2, \mathcal{C}_t, \mathcal{S}_t) \\ \text{s.t.} & \\ C_t = & \ c + c_1 + c_2 \end{aligned} \tag{7}$$

which solves the intra-temporal consumption allocation between public consumption, c, and private consumption of member 1 and 2, c_1 and c_2, respectively, for a given level of total consumption, C_t (and other states).[4]

In unconstrained, consumption solves the first-order condition (see eq. (2))

$$U'(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu) = -w_t(\mathcal{C}_t, \mathcal{S}_t, A_t, \mu)$$

where $U'(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu)$ is the marginal household utility of consumption and the right-hand-side is the discounted expected marginal value of wealth if remaining married,

$$w_t(\mathcal{C}_t, \mathcal{S}_t, A_t, \mu_{t-1}) = \beta \mathbb{E}\left[\frac{\partial \mu_t V^m_{1,t+1}(\mathcal{S}_{t+1}, \mu_t) + (1-\mu_t) V^m_{2,t+1}(\mathcal{S}_{t+1}, \mu_t)}{\partial A_t} \frac{\partial A_t}{\partial C_t} | \mathcal{C}_t, \mathcal{S}_t, A_t, \mu_{t-1}\right].$$

This derivative is generally quite complicated as it involves the derivative of the endogenous bargaining power, μ_t, wrt. wealth. In our example below, we will calculate this object numerically using finite differences.

Let $\overrightarrow{A}_t$ be a grid of end-of-period wealth. Optimal total consumption can then be found as

$$\overrightarrow{C}_t = U'^{-1}(w(\mathcal{C}_t, \mathcal{S}_t, \overrightarrow{A}_t, \mu), \mathcal{C}_t, \mathcal{S}_t, \mu) \quad (8)$$

where $U'^{-1}(\bullet)$ is the inverse marginal household utility. Consumption can be expressed as a function of resources, M_t, using the identity

$$\overrightarrow{M}_t = \overrightarrow{C}_t + \overrightarrow{A}_t.$$

Here, we follow the standard assumption that $\mathcal{C}_t$ does not matter for the continuation value and therefore A_t is a "sufficient statistic" for optimal consumption allocation in a given period. For more implementation details, see Carroll (2006); Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017).

With optimal total consumption in hand, the consumption allocation can be solved from (7) and optimal $\mathcal{C}_t$ can be found in an outer search.

Construction of the inverse marginal household utility. The exposition above requires an evaluation of the *inverse marginal household utility function* to invert the first order condition. In all existing applications of the EGM, this function is known in a closed form. In the current setting, however, this is very unlikely due to the potential non-separability

[4]We assume throughout that there is no dynamic effects of consumption composition itself when focusing on the choice of a couple, conditional on them remaining together.

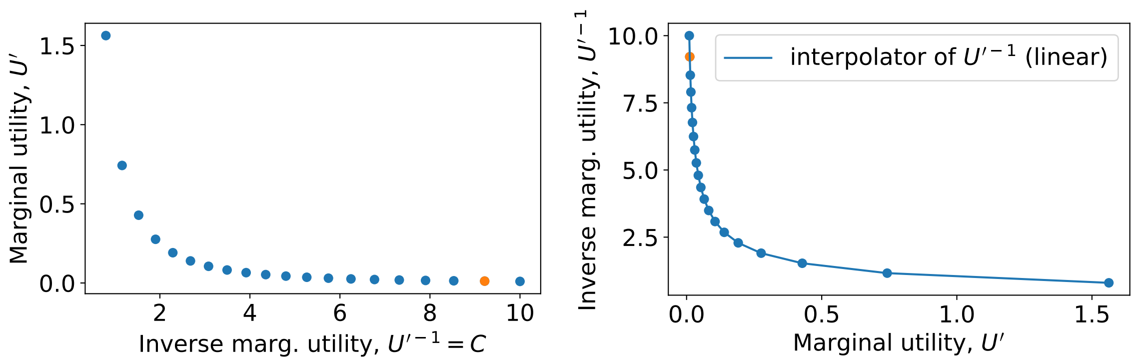
between C_t and $\mathcal{C}_t$ along with both private and public consumption included in total consumption, C_t. We now describe an easy way to construct an interpolator of the key element in the EGM, namely the inverse marginal utility, U'^{-1} proposed in Hallengreen, Jørgensen and Olesen (2024). While the approach applies even when U'^{-1} is not known analytically, a necessary requirement is that it exists and is unique. See Druedahl and Jørgensen (2017) for a discussion of the model requirements for the EGM to be applicable.

We propose to construct $U(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu)$ on a grid of $(\overrightarrow{C}_t, \overrightarrow{\mathcal{C}_t}, \overrightarrow{\mathcal{S}_t}, \overrightarrow{\mu})$, referred to as $\overrightarrow{U}$, and use interpolation to solve this intra-temporal problem fast. Importantly, depending on the exact utility function specification, this object is likely to be a relatively low-dimensional object. Furthermore, this problem only needs to be solved once. Similarly, we propose to construct an interpolator for the marginal utility of consumption, $U'(C_t, \mathcal{C}_t, \mathcal{S}_t, \mu)$. This can be constructed e.g. by finite differences on the grid of $(\overrightarrow{C}_t, \overrightarrow{\mathcal{C}_t}, \overrightarrow{\mathcal{S}_t}, \overrightarrow{\mu})$ if not known analytically.

Finally, we propose to follow Hallengreen, Jørgensen and Olesen (2024) and construct an interpolator for the inverse of the marginal utility of consumption, $U'^{-1}(U, \mathcal{C}_t, \mathcal{S}_t, \mu)$, which is equal to total consumption cf. (8). This can be done by first constructing a grid over total consumption, $\overrightarrow{C}$, and evaluating the marginal utility (just constructed) for all points on this grid to obtain a grid of marginal utilities $\overrightarrow{U'} = U'(\overrightarrow{C}, \mathcal{C}_t, \mathcal{S}_t, \mu)$. Evaluating the inverse marginal utility at some point u', $C(u') = U'^{-1}(u', \mathcal{C}_t, \mathcal{S}_t, \mu)$, can now be done by (linear) interpolation using that $\overrightarrow{C}$ is known for the points in $\overrightarrow{U'}$ just calculated. The marginal utility of consumption typically depends on relatively few states and choices, and the interpolator is only constructed once. Figure 1 illustrates this graphically.

Figure 1: Interpolator of the Inverse Marginal Utility, $\check{C}$ utility.].

(a) Grid over $\overrightarrow{U'^{-1}} = \overrightarrow{C}$ gives $\overrightarrow{U'}$. (b) Flipped Axis and Interpolator, $\check{C}(U')$.



Notes: The figure illustrates in panel a how constructing a grid over values of the inverse marginal utility (or consumption) $\overrightarrow{U'^{-1}} = \overrightarrow{C}$ can be used to evaluate the marginal utility, $\overrightarrow{U'}$. Panel b shows how flipping the axes provides an interpolator for the object of interest, namely $\check{C}$ using the known points in $(\overrightarrow{U'}, \overrightarrow{C})$. The orange-colored marker highlights the flip of axes.

Checking the participation constraints and solving for $\mu^\star$. To check the participation constraints, we calculate the value of remaining a couple, $V_t^{m\to m}(\mathcal{S}_t, \overrightarrow{\mu})$, for all values of bargaining powers, $\overrightarrow{\mu}$. The participation constraint can be checked, and the bargaining process implemented, by following the steps below for each point in the state space, $\mathcal{S}_t$ with the outside option of transitioning to singlehood, $V_{j,t}^{m\to s}(\mathcal{S}_{j,t})$ also calculated.

1. Calculate the marital surplus for each household member for each value in $\overrightarrow{\mu}$ from eq. (4). Denote the resulting arrays as $\overrightarrow{s_1}$ and $\overrightarrow{s_2}$. These arrays are, respectively, ascending and descending grids (in μ).

2. Check if bargaining is relevant for any μ.

   (a) If $\min(\overrightarrow{s_j}) < 0$ and $\max(\overrightarrow{s_j}) < 0$ for any $j \in \{1,2\}$ there is no room for bargaining as the surplus is negative for all values of μ. In turn, they divorce and all values for entering a period as a couple are updated with values associated with being single. Figure 2, panel (a) displays this case.

   (b) If $\min(\overrightarrow{s_1}) \geq 0$ and $\min(\overrightarrow{s_2}) \geq 0$ both members will remain together independent of the bargaining power. In turn, all values for entering a period as a couple are updated to the solutions to the "remaining couple" problem in eq. (3). Figure 2, panel (b) displays this case.

3. Terminate if one of the cases is true in step 2 above. If not, find the indifference points, where each member is just indifferent between remaining and divorcing. We

denote these points by $\tilde{\mu}_1$ and $\tilde{\mu}_2$, respectively. These points are found by linear interpolation, as illustrated in Figure 3, panel (a):

(a) Find the lowest (highest) point in the grid $\overrightarrow{\mu}$ that makes member 1 (member 2) satisfied with remaining married,

$$i_1 : \overrightarrow{s_1}[i_1] \geq 0, \overrightarrow{s_1}[i_1 - 1] < 0$$
$$i_2 : \overrightarrow{s_2}[i_2 + 1] < 0, \overrightarrow{s_2}[i_2] \geq 0$$

where an index in square brackets indicates an element of the array.

(b) Use linear interpolation to approximate the point of indifference. Concretely, imagine that member 1 is unsatisfied with the current bargaining power. We then set member 1's surplus to zero using linear interpolation,

$$0 = \overrightarrow{s_1}[i_1 - 1] + \frac{\overrightarrow{s_1}[i_1] - \overrightarrow{s_1}[i_1 - 1]}{\overrightarrow{\mu}[i_1] - \overrightarrow{\mu}[i_1 - 1]}(\tilde{\mu}_1 - \overrightarrow{\mu}[i_1 - 1])$$

by isolating

$$\tilde{\mu}_1 = \overrightarrow{\mu}[i_1 - 1] - \overrightarrow{s_1}[i_1 - 1]\frac{\overrightarrow{\mu}[i_1] - \overrightarrow{\mu}[i_1 - 1]}{\overrightarrow{s_1}[i_1] - \overrightarrow{s_1}[i_1 - 1]}.$$

Similarly, for member 2, the indifference point is

$$\tilde{\mu}_2 = \overrightarrow{\mu}[i_2] - \overrightarrow{s_2}[i_2]\frac{\overrightarrow{\mu}[i_2 + 1] - \overrightarrow{\mu}[i_2]}{\overrightarrow{s_2}[i_2 + 1] - \overrightarrow{s_2}[i_2]}.$$

The situation in which one member will always remain while the other member has a point of indifference is also a possibility. In such a situation, the indifference point of the member always willing to remain can be set to be outside the grid.

4. Check if bargaining is possible. If the indifference point of member 2 is lower than that of member 1, $\tilde{\mu}_1 > \tilde{\mu}_2$, there is no room for bargaining and the couple divorces. All values associated with entering as a couple are thus set to that of singlehood. Figure 2, panel (c) displays this case.

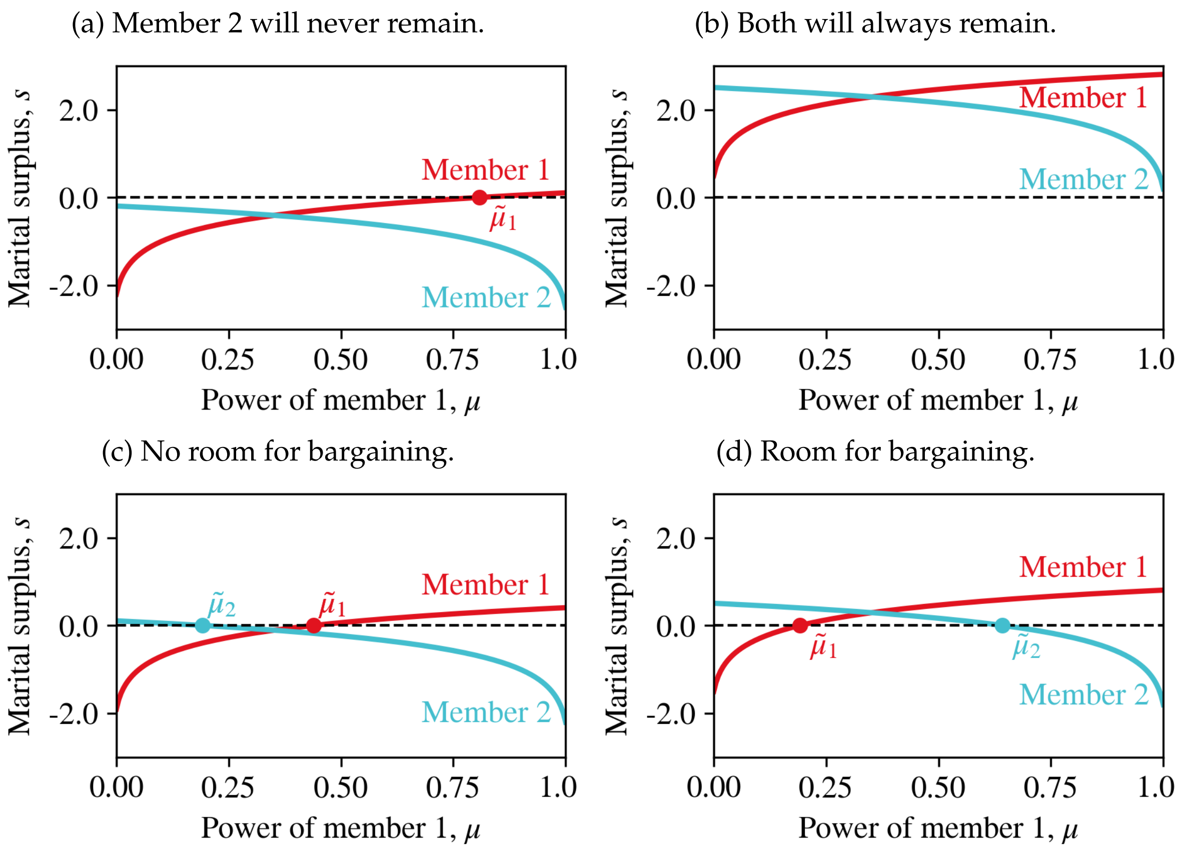
5. If bargaining is possible in step 4 above, determine the value of the updated bargaining power, $\mu_t^\star = \mu_t$ depending on the beginning-of-period bargaining state, μ_{t-1} in $\overrightarrow{\mu}$. This case is illustrated in panel (d) in Figure 2 and the bargaining process is illustrated in Figure 3:

(a) If $\tilde{\mu}_1 \leq \mu_{t-1} \leq \tilde{\mu}_2$, they remain together with unchanged bargaining power, $\mu_t = \mu_{t-1}$, and all values for entering a period as a couple are updated to the solutions to the conditional problem.

(b) If $\mu_{t-1} \leq \tilde{\mu}_1$, set $\mu_t = \tilde{\mu}_1$ and interpolate all objects to values at $\tilde{\mu}_1$. Note that this can be done once since for all $\mu_{t-1} < \tilde{\mu}_1$ the bargaining power will be updated to $\tilde{\mu}_1$ and all functions will be identical for all values of $\mu < \tilde{\mu}_1$ in $\overrightarrow{\mu}$.

(c) Similarly, if $\tilde{\mu}_2 < \mu_{t-1}$, set $\mu_t = \tilde{\mu}_2$ and interpolate all objects to the values at $\tilde{\mu}_2$. Again, for all $\mu > \tilde{\mu}_2$, the solution will be identical.

Figure 2: Bargaining Cases.

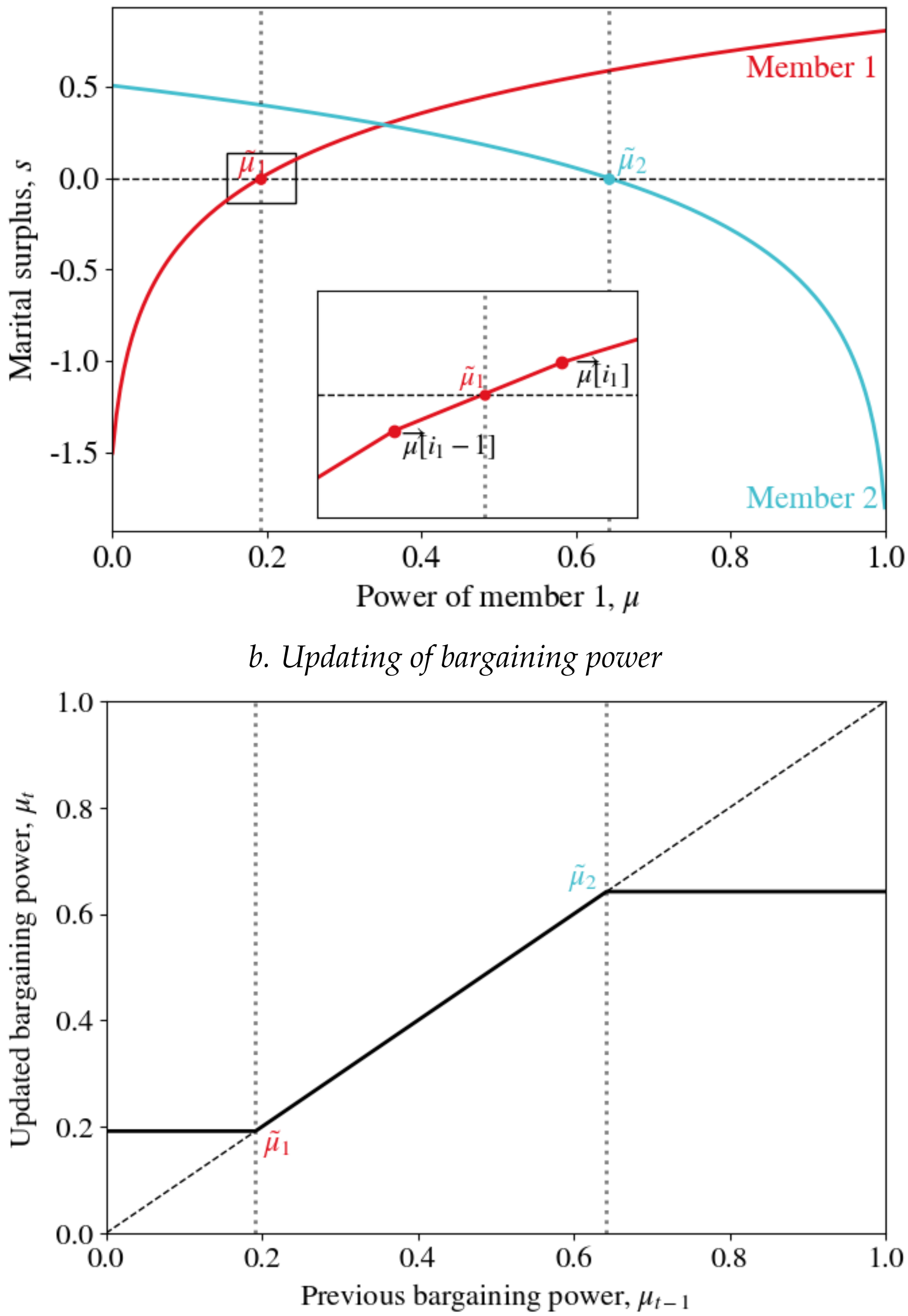


Notes: The figure illustrates the marital surplus for member 1 and 2, $\overrightarrow{s_1}$ and $\overrightarrow{s_2}$. In panel (a) member 2 does not have a positive marital surplus for any bargaining power, μ, so the household divorces. In panel (b) both members have a positive marital surplus for any bargaining power, so they will remain together and will not renegotiate the bargaining power. In panel (c) there exists for each member a bargaining power that result in a positive marital surplus, but not a bargaining power that makes both members have a positive marital surplus. The members will not be able to settle on a bargaining power, so they will divorce. In panel (d) there exists a bargaining power that results in a positive marital surplus for both members at the same time. The members will remain together with a bargaining power from the set of bargaining powers that result in a positive marital surplus for both members.

Figure 3: Bargaining algorithm

a. Interpolation of the Indifference Point

b. Updating of bargaining power



Notes: Panel a illustrates how the indifference point is found by linear interpolation of member 1's the marital surplus function. The interpolation is done between the grid point that makes member 1 satisfied with remaining married, i, and the point before that, $i - 1$. The interpolated indifference point, $\tilde{\mu}_1$, is where the martial surplus is zero. Panel b illustrates how the bargaining power is updated.

4 Example: Dynamic consumption allocation

In this section, we present a model of dynamic consumption allocations with limited commitment bargaining within a household. The household consists of a woman and a man, indexed w and m, respectively. The couple bargains according to the algorithm described in section 2 and split up if an agreement cannot be reached. Single individuals participate in the marriage market and can remarry if they meet a suitable partner. The full dynamics of the couples' and singles' problems, respectively, are described in the following.

In this example, we set up a model where couples choose individual consumption, $c_{j,t}$ to $j \in \{w, m\}$, and public consumption, c_t. We have three state variables: beginning of period t wealth, A_{t-1}, match quality, ψ_t and the bargaining power coming into the period, μ_{t-1}. From the previous notation, this corresponds to $\mathcal{S}_t = (\psi_t, A_{t-1})$.

Individual preferences are of the CES type,

$$U_j(c_{j,t}, c_t) = \frac{1}{1-\rho_j}\left(\alpha_{1,j}c_{j,t}^{\phi_j} + \alpha_{2,j}c_t^{\phi_j}\right)^{1-\rho_j} \tag{9}$$

and the budget constraint for a couple is

$$A_t + c_t + c_{w,t} + c_{m,t} = RA_{t-1} + Y_{w,t} + Y_{m,t}, \quad A_t \geq 0$$

where R is the gross interest rate and $Y_{j,t}$ is exogenous income of member j. The household utility function is a weighted sum of individual utilities with the weight μ on the woman's utility. Couples also receive utility from match quality, ψ_t, which enters additively in the value function. Match quality follows a unit root process:

$$\psi_{t+1} = \psi_t + \varepsilon_{t+1}$$

where $\varepsilon \sim iid\mathcal{N}(0, \sigma_\psi^2)$. This "love shock" is the only source of uncertainty for couples.

Single individuals also choose individual consumption, $c_{j,t}$ and public consumption, c_t. The state variable for singles is $\mathcal{S}_{j,t} = (A_{j,t-1})$, since singles do not engage in bargaining or have a match quality. Individual preferences are still described by (9). Singles face the budget constraint

$$A_{j,t} + c_t + c_{j,t} = RA_{j,t-1} + Y_{j,t}, \quad A_{j,t} > 0$$

The value of remaining single is

$$V_{j,t}^{s\to s}(A_{j,t-1}) = \max_{c_{j,t},c_t} U_j(c_{j,t}, c_t) + \beta\mathbb{E}_t[V_{j,t+1}^{s}(A_{j,t}, A_{j,t}^{p}, \psi_{t+1})]$$

where $\mathbb{E}_t[V_{j,t+1}^{s}(A_{j,t}, A_{j,t}^{p}, \psi_{t+1})]$ denotes the expected value of entering period $t+1$ as single (described below).

The expected value of entering a period as single is comprised of the value of meeting a partner, which happens with probability p_t, and the value of staying single:

$$\mathbb{E}_t[V_{j,t+1}^{s}(A_{j,t}, A_{j,t}^{p}, \psi_{t+1})] = p_t\mathbb{E}_t\left[\tilde{V}_{j,t+1}(A_{j,t}, A_{j,t}^{p}, \psi_{t+1})\right] + (1-p_t)V_{j,t+1}^{s\to s}(A_{j,t})$$

where $\tilde{V}_{j,t}(A_{j,t-1}, A_{j,t-1}^{p}, \psi_t)$ denotes the value of meeting a partner with assets $A_{j,t-1}^{p}$ and initial match quality ψ_t:

$$\begin{aligned}\tilde{V}_{j,t}^{s}(A_{j,t-1}, A_{t-1}^{p}, \psi_t) &= M_t^{\star}V_{j,t}^{s\to m}(\psi_t, A_{t-1}) + (1-M_t^{\star})V_{j,t}^{s\to s}(A_{j,t-1})\\ &\text{s. t.}\\ A_{t-1} &= A_{j,t-1} + A_{j,t-1}^{p},\end{aligned}$$

where $V_{j,t}^{s\to m}(\psi_t, A_{t-1})$ is the value of transitioning from singlehood to marriage, $V_{j,t}^{s\to s}(A_{j,t-1})$ is the value of remaining single, and $M_t^{\star}$ is the optimal choice to marry or not (defined later).

When taking the expectation of $\tilde{V}_{j,t}(\bullet)$ with respect to the characteristics of the partner, we let the partner's wealth conditional on own wealth, and initial match quality follow the independent distributions $\Gamma_{A_j^p}(a|A_{j,t})$ and $\Gamma_{\psi}(\psi)$. In turn, the expected value is

$$\mathbb{E}_t[\tilde{V}_{j,t}^{s}(A_{j,t-1}, A_{j,t-1}^{p}, \psi_t)] = \int_0^{\infty}\int_{-\infty}^{\infty}\tilde{V}_{j,t}^{s}(A_{j,t-1}, a, \psi)\Gamma_{\psi}(\psi)\Gamma_{A_j^p}(a|A_{j,t-1})d\psi da.$$

The value of transitioning from couple to single is similar to remaining single but couples incur a divorce cost of χ

$$V_{j,t}^{m\to s}(A_{j,t-1}) = V_{j,t}^{s\to s}(A_{j,t-1}) - \chi$$

where all choices thus are identical as those of someone remaining single.

The value of remaining a couple with bargaining power μ is:

$$V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = U_j(\tilde{c}_{j,t}, \tilde{c}_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t+1}^{m}(\psi_{t+1}, A_t, \mu)]$$
$$\text{s.t.}$$
$$A_t = RA_{t-1} + Y_{w,t} + Y_{m,t} - (\tilde{c}_t + \tilde{c}_{w,t} + \tilde{c}_{m,t})$$
$$\psi_{t+1} = \psi_t + \varepsilon_{t+1}$$

where $(\tilde{c}_{w,t}, \tilde{c}_{m,t}, \tilde{c}_t)$ is the optimal consumption allocation conditional on μ. This is determined by solving the couple's optimization problem conditional on remaining together with the level of bargaining power being μ:

$$\begin{aligned} \tilde{c}_{w,t}(\mu), \tilde{c}_{m,t}(\mu), \tilde{c}_t(\mu) = \arg\max_{c_{w,t}, c_{m,t}, c_t} & \mu v_{w,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\ & + (1-\mu) v_{m,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\ \text{s.t.} & \\ A_t &= RA_{t-1} + Y_{w,t} + Y_{m,t} - (c_t + c_{w,t} + c_{m,t}) \\ \psi_{t+1} &= \psi_t + \varepsilon_{t+1},\ \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\psi^2) \end{aligned} \tag{10}$$

where the value-of-choice given some μ is

$$v_{j,t}(\psi_t, A_{t-1}, \mu, c_{w,t}, c_{m,t}, c_t) = U_j(c_{j,t}, c_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t+1}^{m}(\psi_{t+1}, A_t, \mu)] \tag{11}$$

where $V_{j,t+1}^{m}(\bullet)$ denotes the value of entering period $t+1$ as married.

The value of entering a period as a couple is

$$V_{j,t}^{m}(\psi_t, A_{t-1}, \mu_{t-1}) = D_t^{\star} V_{j,t}^{m\to s}(\kappa_j A_{t-1}) + (1 - D_t^{\star}) V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu_t^{\star})$$

where κ_j is the share of household wealth member j gets in case of divorce ($\kappa_w + \kappa_m = 1$). The determination of the bargaining weight $\mu_t^{\star}$ and the choice of divorce, $D_t^{\star}$, are determined as follows.

The bargaining power is updated according to the algorithm described in section 2. For this purpose let

$$S_{j,t}(\psi_t, A_{t-1}, \mu) = V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) - V_{j,t}^{m\to s}(\kappa_j A_{t-1})$$

denote the marital surplus of household member j.

This gives the updating rule

$$\mu_t^\star = \begin{cases} \mu_{t-1} & \text{if} \quad S_{j,t}(\psi_t, A_{t-1}, \mu_{t-1}) \geq 0 \quad \text{for} \quad j \in \{w, m\} \\ \tilde{\mu}_w & \text{if} \quad S_{w,t}(\psi_t, A_{t-1}, \mu_{t-1}) < 0 \quad \text{and} \quad S_{m,t}(\psi_T, A_{t-1}, \tilde{\mu}_w) \geq 0 \\ \tilde{\mu}_m & \text{if} \quad S_{m,t}(\psi_t, A_{t-1}, \mu_{t-1}) < 0 \quad \text{and} \quad S_{w,T}(\psi_T, A_{t-1}, \tilde{\mu}_m) \geq 0 \\ \emptyset & \text{else} \end{cases}$$

where

$$\tilde{\mu}_j = \{\mu : S_{j,t}(\psi_t, A_{t-1}, \mu) = 0\}$$

The divorce indicator $D^\star$ takes the value 1 if a cooperative bargaining outcome cannot be reached and 0 otherwise, that is

$$D^\star = \begin{cases} 1 & \text{if} \quad \mu^\star = \emptyset \\ 0 & \text{else} \end{cases}$$

The value of transitioning from single to couple is

$$V_{j,t}^{s \to m}(\psi_t, A_{t-1}) = V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0)$$

where $V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0)$ is the value of remaining married (described above) with initial bargaining weight determined through Nash bargaining,

$$\begin{aligned} \mu^0 = \arg\max_{\mu} & (V_{w,t}^{m \to m}(\psi_t, A_{t-1}, \mu) - V_{w,t}^{s \to s}(A_{w,t-1})) \\ & \times (V_{m,t}^{m \to m}(\psi_t, A_{t-1}, \mu) - V_{m,t}^{s \to s}(A_{m,t-1})) \end{aligned}$$

If $V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0) - V_{j,t}^{s \to s}(A_{j,t-1}) > 0$ for $j \in \{m, w\}$ they form a couple and $M_t^\star = 1$, otherwise they do not and $M_t^\star = 0$.

4.1 Numerical solution

In this section, we describe how we solve the model and highlight some specific tricks used. For a detailed description of the implementation, we refer to the GitHub repository with the accompanying code used to generate the results here.[5]

[5] https://github.com/ThomasHJorgensen/HouseholdBargainingGuide

4.1.1 Precomputations

Before solving the model, we perform a number of precomputations to minimize the solution time.

Precomputation of intra-period consumption allocation. The problem of consumption choice for couples can be divided into an intertemporal allocation of consumption between periods and an intratemporal allocation of consumption over different consumption goods. Conditional on bargaining power μ and total consumption in a given period, C, the latter amounts to the following time-invariant problem:

$$c_w(\mu, C), c_m(\mu, C), c(\mu, C) = \arg\max_{c_j, c_m, c} \mu U_w(c_w, c) + (1-\mu) U_m(c_m, c)$$
$$st. \quad C = c_w + c_m + c$$

We solve this problem numerically and construct interpolaters, $\check{c}_w(\mu, C), \check{c}_m(\mu, C), \check{c}(\mu, C)$ over pre-specified grids of C and μ.

For singles, the current specification allows us to write the share of consumption spent on each good in closed form. We have

$$c_j^{single}(C) = \left[1 + \left(\frac{a_{2,j}}{a_{1,j}}\right)^{\frac{1}{1-\phi_j}}\right]^{-1} C$$

and the corresponding public consumption is $C - c_j^{single}$.

Precomputation of total consumption interpolator We construct a grid over total consumption, C, and for each grid point, we compute the marginal utility by taking the numerical derivative of the couple's utility function:

$$U(C) = \max_{c_j, c_m, c} \mu U_w(c_w, c) + (1-\mu) U_m(c_m, c)$$
$$st. \quad C = c_w + c_m + c$$

We save this on a grid of marginal utility, U', and then construct an interpolator of C over marginal utility as done by Hallengreen, Jørgensen and Olesen (2024) and described in section 3, $\check{C}(U')$.

4.1.2 Solving the model

We solve the model by iterating backwards, starting in the terminal period T. We do so using the iEGM algorithm described in Hallengreen, Jørgensen and Olesen (2024), hence to iterate from period $t+1$ to period t, we need to know the expected marginal value of entering period t as single,

$$w_{j,t}(A_{j,t}) = \beta \mathbb{E}_t \left[\frac{\partial V^s_{j,t+1}(A_{j,t})}{\partial A_{j,t}} \right]$$

and for couples,

$$w_t(A_t, \mu) = \beta \mathbb{E}_t \left[\mu \frac{\partial V^m_{w,t+1}(\psi, A_t, \mu)}{\partial A_t} + (1-\mu) \frac{\partial V^m_{m,t+1}(\psi, A_t, \mu)}{\partial A_t} \right].$$

We describe the construction of these objects below.

4.1.3 Terminal period

The value of remaining single in the terminal period T is:

$$V^{s \to s}_{j,T}(A_{j,T-1}) = U_j \left(c^{single}_j(C_T), C_T - c^{single}_j(C_T) \right)$$

where total consumption is $C_T = RA_{j,t-1} + Y_{j,t}$, i.e. all resources.

The value of transitioning from marriage to singlehood is identical to the above apart from a divorce cost,

$$V^{m \to s}_{j,T}(A_{j,t-1}) = V^{s \to s}_{j,T}(A_{j,T-1}) - \chi$$

The value of remaining a couple in the terminal period is

$$V^{m \to m}_{j,T}(\psi_T, A_{T-1}, \mu) = U_j(\check{c}_j(C_T, \mu), \check{c}(C_T, \mu)) + \psi_T$$

where total consumption again amounts to all resources, $C_T = RA_{T-1} + Y_{w,T} + Y_{m,T}$. Note that $V^{m \to m}_{j,t}$ is defined for an arbitrary bargaining power μ.

The marital surplus as a function of μ is then

$$S_{j,T}(\psi_T, A_{T-1}, \mu) = V^{m \to m}_{j,T}(\psi_T, A_{T-1}, \mu) - V^{m \to s}_{j,T}(\kappa_j A_{T-1})$$

where κ_j denotes the share of marital assets received by spouse j in the event of divorce.

The value of entering a period as a couple includes both the possibility of remaining married and divorcing, such that

$$V^m_{j,T}(\psi_T, A_{T-1}, \mu_{T-1}) = D^\star_T V^{m\to s}_{j,T}(\kappa_j A_{T-1}) + (1 - D^\star_T) V^{m\to m}_{j,T}(\psi_T, A_{T-1}, \mu^\star_T)$$

This value depends on the outcome of any potential bargaining, $\mu^\star_T$ and divorce $D^\star_T$ which is updated according to the algorithm described in section 3.

Knowing this value, we can precompute the expected marginal value of entering period T. First, we compute the household value of entering period T as married over a grid of post-decision assets, $\overrightarrow{A}$:

$$V^m_T(\psi_T, \overrightarrow{A}, \mu_{T-1}) = \mu_{T-1} V^m_{w,T}(\psi_T, \overrightarrow{A}, \mu_{T-1}) + (1 - \mu_{T-1}) V^m_{m,T}(\psi_T, \overrightarrow{A}, \mu_{T-1})$$

Next, we approximate the marginal value by centered finite differences on the grid $\overrightarrow{A}$. Letting $\overrightarrow{A}[i]$ denote index i on $\overrightarrow{A}$:

$$\frac{\partial V^m_{j,T}(\psi_T, \overrightarrow{A}[i], \mu_{T-1})}{\partial A} = \frac{V^m_{j,T}(\psi_T, \overrightarrow{A}[i+1], \mu_{T-1}) - V^m_{j,T}(\psi_T, \overrightarrow{A}[i-1], \mu_{T-1})}{\overrightarrow{A}[i+1] - \overrightarrow{A}[i-1]}$$

where we extrapolate the slope at the first and last grid points.

Finally, we compute the expected marginal value over a grid of post decision assets, $\overrightarrow{A}$ and post-decision bargaining $\overrightarrow{\mu}$:

$$w_{T-1}(\overrightarrow{A}, \overrightarrow{\mu}) = \beta \sum_{q=1}^{Q} \omega^q \left[\overrightarrow{\mu} \frac{\partial V^m_{w,T}(\psi^q, \overrightarrow{A}, \overrightarrow{\mu})}{\partial A} + (1 - \overrightarrow{\mu}) \frac{\partial V^m_{m,T}(\psi^q, \overrightarrow{A}, \overrightarrow{\mu})}{\partial A} \right]$$

where we use Q Gauss Hermite quadrature nodes to take expectations over future values of match quality ψ_T and interpolate $V'_{j,T}$ using linear interpolation. This allows us to construct an interpolator for the expected marginal value, $\breve{w}_{T-1}(A_{T-1}, \mu_{T-1})$.

The value of starting as single conditional on meeting a partner with assets A^p_{T-1} and initial match quality ψ_T is

$$\tilde{V}^s_{j,t}(A_{j,t-1}, A^p_{t-1}, \psi_t) = M^\star_t V^{s\to m}_{j,t}(\psi_t, A_{t-1}) + (1 - M^\star_t)V^{s\to s}_{j,t}(A_{j,t-1})$$
$$\text{s. t.}$$
$$A_{t-1} = A_{j,t-1} + A^p_{t-1},$$

We precompute initial bargaining power for each combination of own assets and partner's assets, $(A_{j,t}, A_{i,t})$ by first computing repartnering surplus over a grid of bargaining power $\overrightarrow{\mu}$:

$$S^{s\to m}_{j,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \overrightarrow{\mu}) = V^{m\to m}_{j,T}(\psi_t, A_{T-1}, \overrightarrow{\mu}) - V^{s\to s}_{j,T}(A_{j,T-1})$$
$$S^{s\to m}_{i,T}(\psi_T, A_{i,T-1}, A_{j,T-1}, \overrightarrow{\mu}) = V^{m\to m}_{i,T}(\psi_T, A_{T-1}, \overrightarrow{\mu}) - V^{s\to s}_{i,T}(A_{i,T-1})$$
$$\text{st. } A_{T-1} = A_{i,T-1} + A_{j,T-1}$$

We interpolate the values of $V^{m\to m}_{j,T}$ using linear interpolation.

We then determine initial bargaining power:

$$\mu_0(\psi_T, A_{j,T}, A_{i,T}) = \arg\max_{\mu} S^{s\to m}_{j,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \mu) S^{s\to m}_{i,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \mu)$$

The expected value of starting as single is computed as

$$\mathbb{E}_{T-1}\left[V^s_{j,T}(A_{j,T-1})\right] = p_T\left[\sum_k^{K_\psi}\sum_n^{N_A} p^\psi_k p^A_n \tilde{V}^s_{j,T}(A_{j,T-1}, A^p_n, \psi_k)\right] + (1 - p_T)V^{s\to s}_{j,T}(A_{j,T-1})$$

where p^ψ_k denotes the probability of drawing initial match quality ψ_i and $p^A_n = p^A_n(A_{j,T-1})$ denotes the probability of drawing partner's assets A^p_j. These probabilities are specified on a grid of (A_j, A^p_j) to represent discretized approximations of Γ_ψ and $\Gamma_{A^p_j}$.

Finally, much like in the case of couples, we compute the expected marginal value of entering period T as single over a grid of post-decision assets, $\overrightarrow{A}$ using centered finite differences, with:

$$w_{j,T}(\overrightarrow{A}[i]) = \frac{\mathbb{E}_{T-1}\left[V^s_{j,T}(\overrightarrow{A}[i+1])\right] - \mathbb{E}_{T-1}\left[V^s_{j,T}(\overrightarrow{A}[i-1])\right]}{\overrightarrow{A}[i+1] - \overrightarrow{A}[i-1]} \tag{12}$$

We use this to construct an interpolator for the marginal expected value of entering a period as single, $\breve{w}_{j,T-1}(A_{T-1})$.

4.1.4 Earlier periods

Solving earlier periods follows almost the same approach as the terminal period, except that we use EGM to determine consumption.

The value of remaining single is computed using standard EGM. We construct a grid over post-decision assets, $\overrightarrow{A}$. We can then interpolate the expected marginal value of entering period $t+1$ using the interpolator $\breve{w}_{j,t}(A_{j,t})$. We make use of the fact that the marginal utility for singles is analytically invertible to find the total consumption, $C_{j,t}$:

$$C_{j,t}(\overrightarrow{A}_t) = U_j'^{-1}(\breve{w}_{j,t}(\overrightarrow{A}_t))$$

With this, we construct an endogenous grid over resources:

$$\overrightarrow{A}_{j,t-1} = R(C_{j,t}(\overrightarrow{A}_t) + \overrightarrow{A}_t) + Y_{j,t}$$

from which we can now interpolate optimal consumption given beginning of period assets $A_{j,t-1}$. We enforce the credit constraint by setting consumption equal to total resources for all asset values below the first point in the endogenous grid. Consumption is now computed over an endogenous grid. It can be helpful to interpolate consumption back onto a common grid for A used throughout all periods.

This allows us to compute the value of remaining single:

$$\begin{aligned} V_{j,t}^{s\to s}(A_{j,t-1}) &= U_j(c_j^{single}(C_{j,t}), C_{j,t} - c_j^{single}(C_{j,t})) + \beta\mathbb{E}_t\left[V_{j,t+1}^{s}(A_{j,t})\right] \\ \text{st.} \quad A_{j,t} &= RA_{j,t-1} - C_{j,t} + Y_{j,t} \end{aligned}$$

Due to the discrete choice of whether to remarry, the value function for singles may have non-concave regions. We deal with this by taking an upper envelope over decision-specific value functions to determine optimal consumption (see Iskhakov, Jørgensen, Rust and Schjerning, 2017).

Consequently, the computation of the value of transitioning from marriage to singlehood is identical to that of the terminal period.

The value of remaining a couple is similarly computed by interpolating the expected marginal value over a grid of post-decision states, $\overrightarrow{A}$. However, this time, we cannot analytically invert the couple's marginal utility function. Instead, we compute total consumption using the precomputed interpolater:

$$C_t = \check{C}(w_{t+1})$$

From here, we follow the same EGM approach as described above to compute consumption as a function of beginning of period assets. Again, we take an upper envelope over decision-specific valuefunctions to deal with potential non-concave regions stemming from the possibility of divorce. We can then compute the value of remaining a couple with bargaining power μ:

$$V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = U_j(\check{c}_j(C_t), \check{c}(C_t)) + \beta \mathbb{E}_t \left[V_{j,t+1}^{m}(\psi_{t+1}, A_t, \mu) \right]$$

The consecutive steps to compute the expected marginal values of entering period t as a couple w_t, and as single, $w_{j,t}$ follow the approach of the terminal period, and the steps can be iteratively repeated until the initial time period.

4.2 Policy Functions

We solve the model for three different parameterizations; see Table 1. The first column (Model 1) contains the baseline values and the other columns only show deviations from the baseline model.

Couples' consumption is determined by time, power, match quality, and assets. We show consumption as a function of assets in the terminal period for three initial values of bargaining power: one where the man decides almost everything, $\mu_{T-1} = 0.05$; one where the bargaining power is equal, $\mu_{T-1} = 0.5$; and one where the woman decides almost everything, $\mu_{T-1} = 0.95$. We also show how the updated bargaining power, $\mu_T^\star$, as well as the value functions, $V_{j,T}^m$. These are depicted for model 1, cf. table 1, in figure 4.

The woman's private consumption is higher when her bargaining power is higher, and vice versa for the man. When the initial bargaining power is $\mu_{t-1} = 0.05$ or $\mu_{t-1} = 0.95$, the updated bargaining power becomes $\mu_t = 0.23$ and $\mu_t = 0.77$, their respective indifference points. For initial bargaining powers between the indifferent points, e.g. $\mu_t = 0.50$ as displayed, the bargaining power is not updated. In this model, assets are shared equally upon divorce and consequently do not affect the relative value of the outside option of either spouse. This results in bargaining power being constant over assets.

If we turn to model 2 where the woman only gets 23% of the assets upon divorce, an increase in assets will then increase the man's outside option relative to the woman's. Figure 5 displays this case. Whenever the woman's participation constraint is binding, an

Table 1: Parameter Values.

	Model 1	**Model 2**	**Model 3**
Income			
R	1.03		
Y_w	1.0		
Y_m	1.0		
Preferences			
β	$1/R$		
ρ_w	2.0		
ρ_m	2.0		
$\alpha_{1,w}$	1.0		
$\alpha_{1,m}$	1.0		
$\alpha_{2,w}$	1.0		
$\alpha_{2,m}$	1.0		
ϕ_w	0.2		
ϕ_m	0.2		
Household bargaining			
κ_w	0.5	0.23	
κ_m	0.5	0.77	
σ_ψ	0.1		0.0
χ	0.0		
Repartnering			
p_t	0.1		
$\Gamma_\psi(\psi)$	$\mathcal{N}(0, \sigma_\psi)$		
$\Gamma_A(A_j^p \mid A_j)$	Deterministic*		

*In the example, we set A_j^p as deterministic conditional on A_j, such that $A_j^p = A_j$ for all values of A_j.

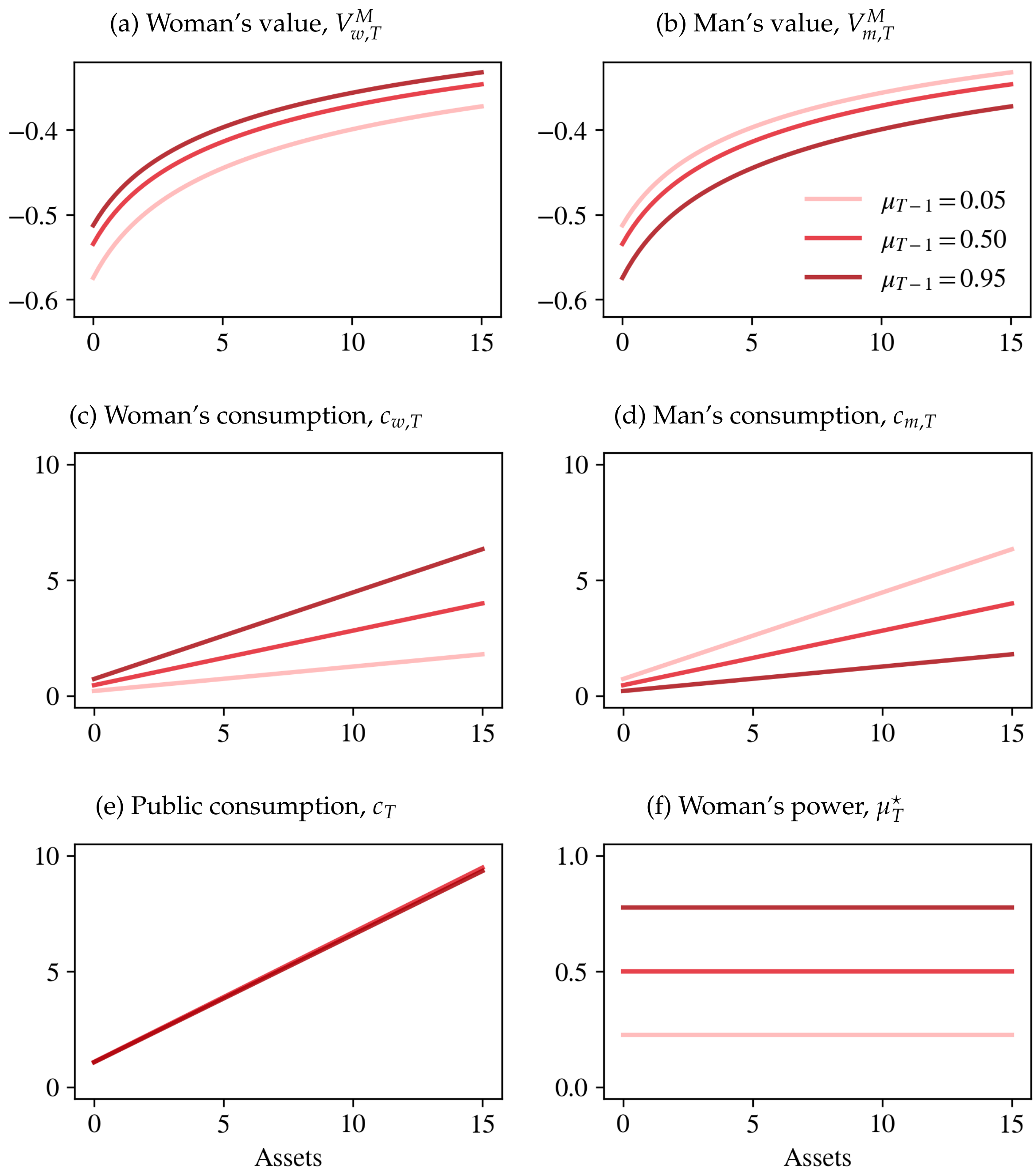
increase in assets affects her private consumption in two ways: 1) She is more wealthy, so she consumes more; and 2) she is relatively worse off when negotiating a new bargaining power, so she consumes less. Both effects are in play in the case where the woman initially has low bargaining power, $\mu_{t-1} = 0.05$. The wealth effect is dominating, but her consumption does not increase as much in assets as in model 1, because her negotiated bargaining power decreases when assets increase.

If her participation constraint is not binding, only the wealth effect plays a role. This is the case for an even bargaining power, $\mu_{t-1} = 0.5$ when assets are low. Here, bargaining power is constant as assets increase from a low level, and consumption increases for both partners due to the wealth effect only. However, when assets exceed approximately 8, the man's participation constraint becomes binding, inducing bargaining, and shifting private consumption from the woman to the man.

For an initial bargaining power of $\mu_{t-1} = 0.95$, it is the man's participation constraint that is binding. As his outside option becomes better relative to the woman's when assets increase, he can negotiate a lower bargaining power for the woman. In this case for high enough assets, he can even negotiate a bargaining power below 50%,.

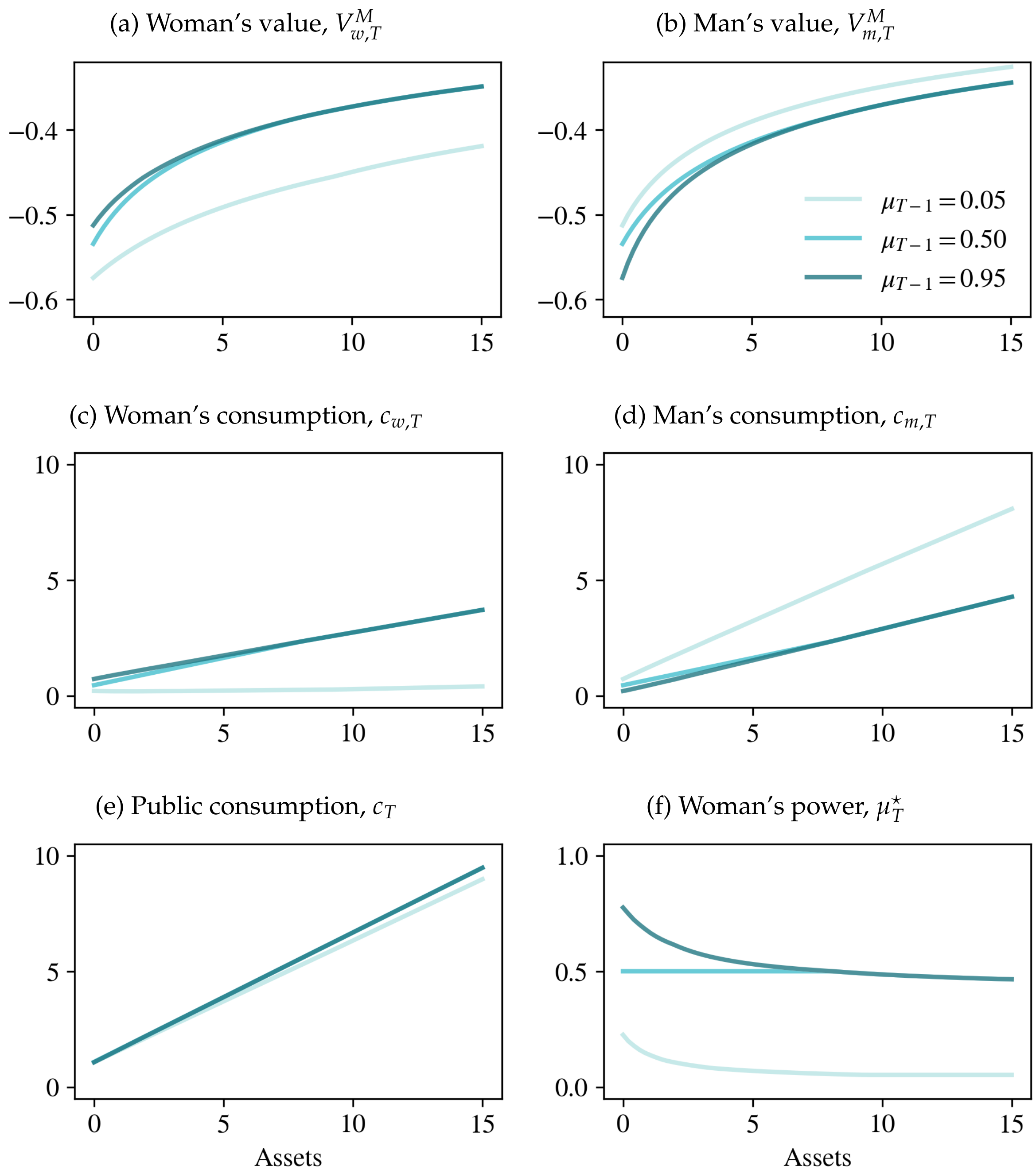
Finally, we take a look at the effect of match quality in model 1 with an even asset split upon divorce. In Figure 6 we show the same variables for the same three initial bargaining powers, but now across match quality for a couple with an asset level of 5. The first thing to notice is that if the couple has sufficiently negative match quality, they will divorce as indicated by the missing updated bargaining power. When the partners divorce and enter the single state, match quality no longer plays a role, so the consumption and value functions are constant. Note that the bargaining power is only updated when the couple has a relatively low value of match quality and thereby is close to divorce. This is because they can make credible threats of leaving when bargaining. For higher values of match quality, their bargaining power will not be updated as they have no credible threat of leaving. When match quality is high, initial bargaining power can therefore have a huge impact on the consumption allocation between the man and the woman.

Figure 4: Model 1, $\sigma_\psi = 0.1$, equal asset split



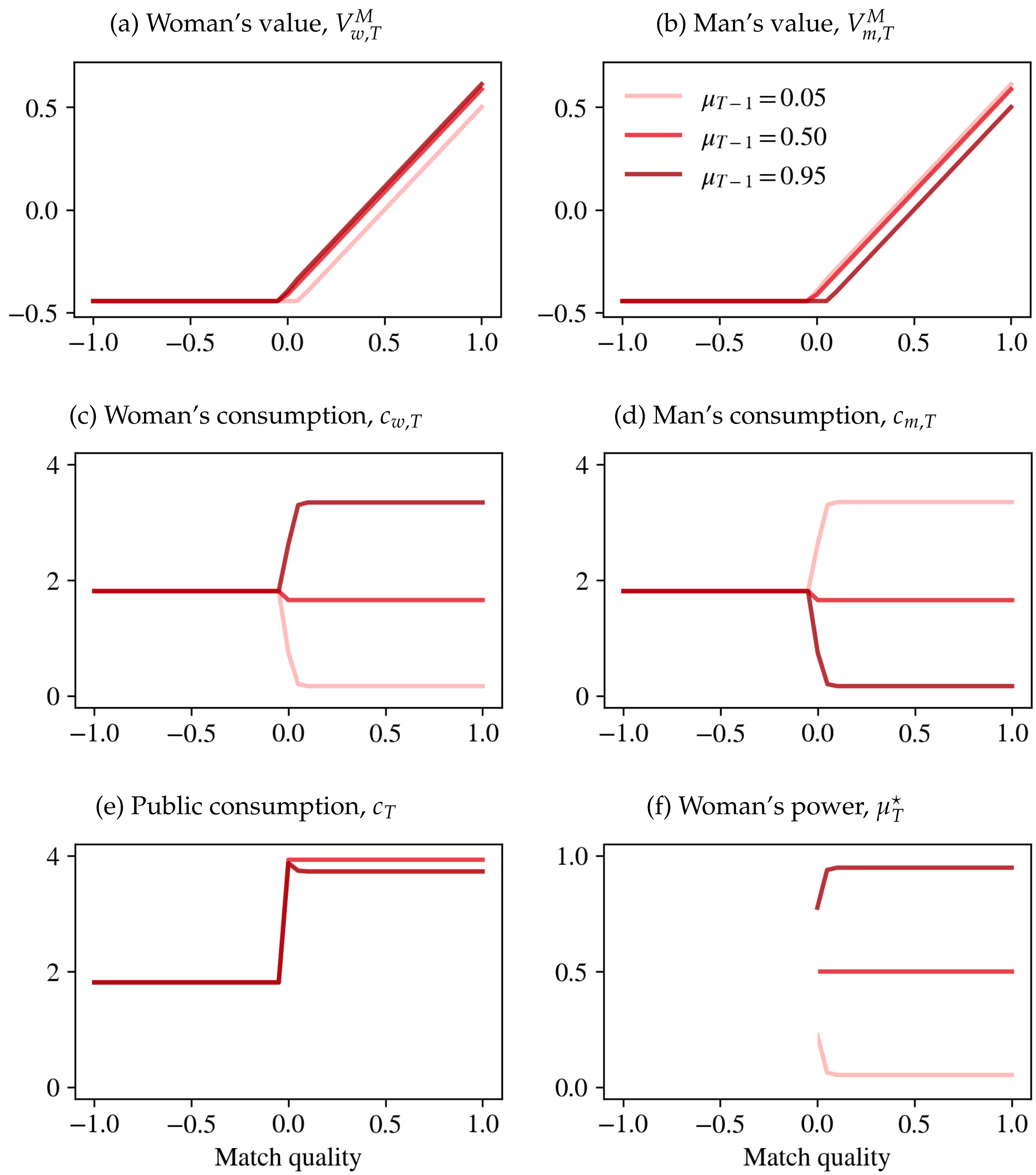
*Notes:*The figure contains value and policy functions for model 1, where match quality is stochastic and assets are split equally upon divorce. The figure illustrates the value of entering period $T = 10$ as married for the woman and the man respectively (first row), the woman's and man's private consumption in period (second row), and public consumption and the woman's updated bargaining power in period (third row), all for the terminal period T, all as a function of beginning-of-period assets, A_{T-1}. The dotted lines show the case where the couple enters the period with the woman's bargaining power $\mu_{T-1} = 0.05$, the dashed lines show the case where $\mu_{T-1} = 0.5$, and the fully drawn lines show the case where $\mu_{T-1} = 0.95$.

Figure 5: Model 2, $\sigma_\psi = 0.1$, unequal asset split



Notes: The figure contains value and policy functions for model 2, where match quality is stochastic and assets upon divorce are split with 23% going to the woman and 77% going to the man. The figure illustrates the value of entering period $T = 10$ as married for the woman and the man respectively (first row), the woman's and man's private consumption in period (second row), and public consumption and the woman's updated bargaining power in period (third row), all for the terminal period T, all as a function of beginning-of-period assets, A_{T-1}. The dotted lines show the case where the couple enters the period with the woman's bargaining power $\mu_{T-1} = 0.05$, the dashed lines show the case where $\mu_{T-1} = 0.5$, and the fully drawn lines show the case where $\mu_{T-1} = 0.95$.

Figure 6: Model 1, $\sigma_\psi = 0.1$, equal asset split



Notes: The figure contains value and policy functions for model 1, where match quality is stochastic and assets are split equally upon divorce. The figure illustrates the value of entering period $T = 10$ as married for the woman and the man respectively (first row), the woman's and man's private consumption in period (second row), and public consumption and woman's updated bargaining power in period (third row), all for the terminal period T, all as a function of match quality, ψ_T. The missing lines in panel (f) indicate that the couple divorces. The dotted lines show the case where the couple enters the period with the woman's bargaining power $\mu_{T-1} = 0.05$, the dashed lines show the case where $\mu_{T-1} = 0.5$, and the fully drawn lines show the case where $\mu_{T-1} = 0.95$.

4.3 Model simulations

We simulate 10,000 women over 10 periods from three different models: The baseline model 1 ($\kappa_w = 0.50, \sigma = 0.1$), model 2 where the woman gets 23% of assets upon divorce ($\kappa_w = 0.23, \sigma = 0.1$), and model 3 where the match quality stays constant at zero ($\kappa_w = 0.5, \sigma = 0.0$). We initiate all women in a couple with a match quality of 0, an asset level of 1, and a low bargaining power at 0.02. If the couple splits, we follow the women as they become single and potentially remarry. Figures 7 and **??** show the average simulated behavior over time for couples. This includes couples who have been together throughout the period, as well as new couples that form during the simulation period.

Consider first model 1 (red line). As all couples were initiated on the verge of divorce, couples that receive a negative shock to match quality will likely divorce. This results in a large share of couples splitting during the first periods. The average match quality increases over time due to a selection effect, where couples that receive negative shocks drop out of the sample. Meanwhile, single women select into new marriages based on the match quality with their new partners, which further contributes to the increase in average match quality among couples.

Stochastic shocks to match quality create a risk of divorce. The couple insures themselves by saving at the beginning of the life cycle, creating a slightly hump-shaped wealth profile. Consequently, consumption is lower in the first periods and higher in the later periods.

In each couple, the woman is initiated with such low bargaining power that she wants to leave the marriage. The bargaining process ensures that her bargaining power is updated to the power that makes her indifferent between staying or divorcing. This still gives her less power than her partner, who consequently can leverage his bargaining power to get a higher private consumption than her.

Over time, women's average bargaining power increases. This is mainly driven by remarried women who negotiate a relatively high initial bargaining power. As women's average bargaining power increases, so does their private consumption.

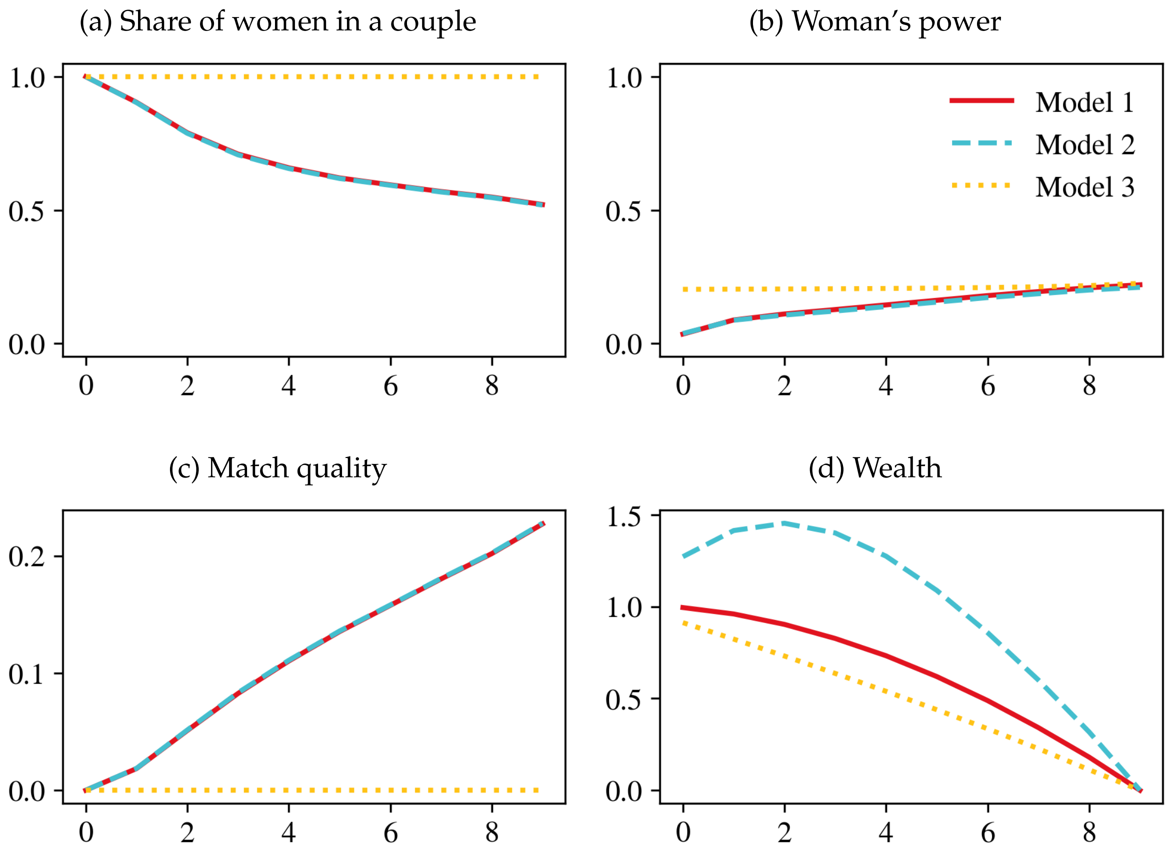
Next, consider model 2 where the asset split upon divorce is unequal in favor of the man (blue line). Despite the woman's outside option being significantly worsened compared to model 1, the share of couples is identical, and the women's average bargaining power is very similar between the two models. This shows that the asset split upon divorce has little effect on bargaining and divorce in this model. Despite that, the couple now saves more and consumes less compared to model 1. This is because the man is more interested in accumulating wealth as he gets to keep most of the assets. Because his bargaining power is relatively high, the couple adheres to his preferences and saves a lot.

This shows that misaligned incentives are important for outcomes within the marriage when the bargaining power is unequal.

Consider finally model 3, where there is no uncertainty about match quality and assets again are split equally upon divorce (yellow line). In the first period, all couples prefer to stay together with an updated bargaining power of 0.23, and since nothing changes over time, they will remain together throughout all periods with this bargaining power. As each couple knows this at the beginning of the life cycle, there is no insurance motive for saving, and they will consume a constant share of their wealth. The woman will consume less than her partner since she has a low bargaining power.

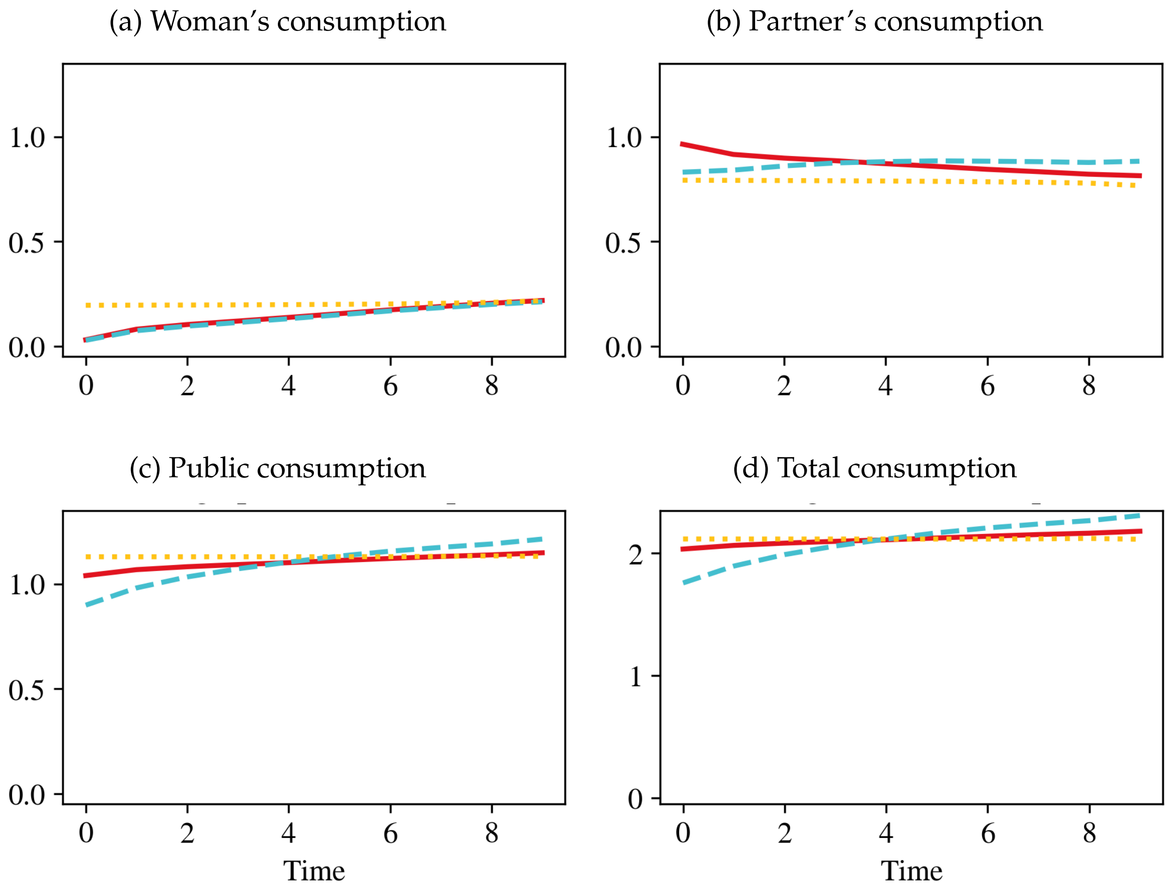
While divorce and bargaining outcomes were largely identical between models 1 and 2, they differ in model 3. This shows that in this model, match quality is more important in determining these outcomes than an unequal asset split upon divorce.

Figure 7: Average simulated states for couples



Notes: The figure contains simulated averages of couples' states and choices for model 1, 2, and 3. The simulation is based on 10,000 women over 10 periods, where all women are initialized in couples with a quality of match of 0, an asset level of 1, and a low bargaining power at 0.02. Panel (a) displays the share of women in a couple for each period. The remaining panels show the average value of the state or choice for couples only, which means that the sample changes over time as the couples split. Panel (b) displays the average updated bargaining power; panel (c) displays the average match quality at the beginning of the period; and panel (d) displays the average asset level at the end of the period.

Figure 8: Average simulated consumption for couples



Notes: The figure contains simulated averages of couples' states and choices for model 1, 2, and 3. The simulation is based on 10,000 women over 10 periods, where all women are initialized in couples with a quality of match of 0, an asset level of 1, and a low bargaining power at 0.02. The panels show the average value of consumption for couples only, which means that the sample changes over time as the couples split. Panel (a) displays women's average consumption; panel (b) displays men's average consumption; panel (c) displays average public consumption; and panel (d) displays total consumption. Note that the single women's ex-partner is replaced by the women's new partner when they remarry.

5 Concluding Discussion

We provide a general and flexible notation that can facilitate the formulation of dynamic household models. Using this notation, we show how the bargaining process in the limited commitment model (see e.g. Mazzocco, 2007) can be expressed as an updating rule of a co-state variable that is the relative bargaining power of each household member. We also provide a discussion on how to efficiently solve this class of models using the endogenous grid method (EGM), proposed by Carroll (2006) and extended by e.g. Iskhakov, Jørgensen, Rust and Schjerning (2017); Druedahl and Jørgensen (2017) by the approach proposed in Hallengreen, Jørgensen and Olesen (2024). We finally show how the bargaining process can be resolved via interpolation of indifference points to further speed up the solution.

Through an example of intra-household consumption allocation, we illustrate the proposed methods and provide Python and C++ code that can easily be adapted to other use cases.

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Supplementary Material

A Recursive Formulation

To derive the recursive formulation of a couple, we follow the exposition in Marcet and Marimon (2019), but adopt our own notation. Let $V_{j,t}^{m\to s}(\mathcal{S}_t)$ denote the value of transitioning from marriage to singlehood (divorce) of household member j as a function of state variables in $\mathcal{S}_t$. Note that this set contains all relevant state variables in excess of marital status (which we will denote with superscripts) and the states measuring the bargaining power of each household member (as we will derive below). Given the set of state variables, couples choose the vector $\mathcal{C}_t$. We assume that the states transition following some known distribution,

$$\mathcal{S}_{t+1} \sim \Gamma(\mathcal{S}_t, \mathcal{C}_t)$$

with $\mathcal{S}_0$ given, and that choices potentially have to satisfy some additional constraints, such as a budget constraint. The important complication in the limited commitment framework is the presence of forward-looking participation constraints, and thus we will ignore all other constraints in the following exposition. In the example below, we will include all constraints in the formulation to be precise.

The Pareto problem of a newly formed couple in period zero can be formulated as

$$\max_{\{\mathcal{C}_t\}_0^T} \lambda_{1,0}\mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t U_1(\mathcal{C}_t,\mathcal{S}_t)\right] + \lambda_{2,0}\mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t U_2(\mathcal{C}_t,\mathcal{S}_t)\right]$$

s.t.

$$\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\beta^\tau U_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right] \geq V_{j,t}^{m\to s}(\mathcal{S}_t) \text{ for } t=0,\ldots,T \text{ and } j=1,2 \qquad \text{(A.1)}$$

where $\mathbb{E}_t[\bullet] = \mathbb{E}[\bullet|\mathcal{S}_t,\mathcal{C}_t]$ denotes a conditional expectation, $\lambda_{1,0}$ and $\lambda_{2,0}$ are the initial weight put on the expected discounted utility of the first and second household member, respectively, and (A.1) are the forward looking marital participation constraints.[6] These constraints ensure that each household member finds it optimal to remain in a couple. If, in a given period, there is no allocation $\mathcal{C}_t$ that can satisfy these constraints, the couple will transition to singlehood. The key here is that couples cannot commit to future actions,

[6]Note the slight difference in the sum in the participation constraint compared to e.q. (3) in Marcet and Marimon (2019). This difference leads to a slight difference in the timing: In the current formulation, the bargaining weight is updated in the current period as a consequence of a participation constraint being binding.

they cannot transfer utility to other household members, and there is unilateral divorce.

The Lagrangian

Again ignoring all other constraints than the forward looking ones in (A.1), the Lagrangian can be formulated as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\mathcal{C}},\gamma,\boldsymbol{\lambda},\mathcal{S}_0) =& \mathbb{E}_0\Bigg[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\Bigg\{\lambda_{j,0}U_j(\mathcal{C}_t,\mathcal{S}_t)+ \\ & + \gamma_{j,t}\left(\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\beta^\tau U_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right] - V_{j,t}^{m\to s}(\mathcal{S}_t)\right)\Bigg\}\Bigg]\end{aligned}$$

with $\boldsymbol{\mathcal{C}} = (\mathcal{C}_0,\mathcal{C}_1,\ldots,\mathcal{C}_T)$, $\boldsymbol{\lambda} = (\lambda_{1,0},\lambda_{2,0})$ and the associated shadow prices of the forward looking constraints in $\gamma = (\gamma_{1,0},\gamma_{1,1},\ldots,\gamma_{1,T},\gamma_{2,0},\gamma_{2,1},\ldots,\gamma_{2,T})$.[7]

The Lagrangian can be greatly simplified. First, due to the law of iterated expectations, the inner expectation can be dropped, i.e., we have that[8]

$$\mathbb{E}_0\left[\gamma_{j,t}\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\beta^\tau U_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right]\right] = \mathbb{E}_0\left[\gamma_{j,t}\sum_{\tau=0}^{T-t}\beta^\tau U_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right]$$

and we can write the Lagrangian as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\mathcal{C}},\gamma,\boldsymbol{\lambda},\mathcal{S}_0) =& \mathbb{E}_0\Bigg[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\Bigg\{\lambda_{j,0}U_j(\mathcal{C}_t,\mathcal{S}_t)+ \\ & + \gamma_{j,t}\left(\sum_{\tau=0}^{T-t}\beta^\tau U_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau}) - V_{j,t}^{m\to s}(\mathcal{S}_t)\right)\Bigg\}\Bigg].\end{aligned}$$

Second, we can collect terms to eliminate the double sum. To see how the same period utility for member j enters several times, think of some time-period $\bar{t}$ and write out the elements in the expectation as

$$\beta^{\bar{t}}\lambda_{j,0}U_j(\mathcal{C}_{\bar{t}},\mathcal{S}_{\bar{t}}) + \beta^{\bar{t}}\gamma_{j,\bar{t}}\sum_{\tau=0}^{T-\bar{t}}\beta^\tau U_j(\mathcal{C}_{\bar{t}+\tau},\mathcal{S}_{\bar{t}+\tau}) - \beta^{\bar{t}}\gamma_{j,\bar{t}}V_{j,\bar{t}}^{m\to s}(\mathcal{S}_{\bar{t}})$$

[7]Since the elements in γ are within a conditional expectation, conditional on information in period zero, γ is *normalized* shadow prices, based on the expected path of state variables. Chiappori and Mazzocco (2017) explicitly account for this in their formulation.

[8]because $\gamma_{j,t}$ is a function of information at time t it can be included in the inner conditional expectation.

which means that $U_j(\mathcal{C}_{\bar{t}}, \mathcal{S}_{\bar{t}})$ will be multiplied by

$$\beta^{\bar{t}}\lambda_{j,0} + \beta^{\bar{t}}\gamma_{j,\bar{t}} + \beta^{\bar{t}-1}\beta\gamma_{j,\bar{t}-1} + \beta^{\bar{t}-2}\beta^2\gamma_{j,\bar{t}-2} + \cdots + \beta^0\beta^{\bar{t}}\gamma_{j,0} = \beta^{\bar{t}}\underbrace{\left[\lambda_{j,0} + \sum_{\tau=0}^{\bar{t}}\gamma_{j,\bar{t}-\tau}\right]}_{\text{call this } M_{j,\bar{t}}}.$$

We can thus finally write the Lagrangian as

$$\mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0) = \mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\left\{M_{j,t}U_j(\mathcal{C}_t, \mathcal{S}_t) - \gamma_{j,t}V_{j,t}^{m\to s}(\mathcal{S}_t)\right\}\right] \tag{A.2}$$

with a recursive formulation for the weights (often referred to as co-states) on current period utility as

$$\begin{aligned} M_{j,t} &= M_{j,t-1} + \gamma_{j,t} \\ M_{j,-1} &= \lambda_{j,0} \end{aligned} \tag{A.3}$$

for $j = 1, 2$. The formulation in eq. (A.2) is similar to that in Mazzocco (2007) and Attanasio and Ríos-Rull (2000).[9] Concretely, the formulation could be expressed in terms of surpluses

$$\mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0) = \mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\left\{M_{j,t-1}U_j(\mathcal{C}_t, \mathcal{S}_t) + \gamma_{j,t}(U_j(\mathcal{C}_t, \mathcal{S}_t) - V_{j,t}^{m\to s}(\mathcal{S}_t))\right\}\right]$$

as in Attanasio and Ríos-Rull (2000).

As noted in e.g. Attanasio and Ríos-Rull (2000), the first-order conditions w.r.t. individual consumption (assumed here to be the first input in the utility function) yields

$$\frac{U_1^1(\mathcal{C}_t, \mathcal{S}_t)}{U_2^1(\mathcal{C}_t, \mathcal{S}_t)} = \frac{M_{2,t}}{M_{1,t}}$$

showing that the weights must be equal (in the interior) if the marginal utilities of consumption are identical. If, say, member 1 has a higher marginal utility from consumption, that must be associated with a relatively lower bargaining power of member 1.

[9] The option value is not explicitly discounted in Mazzocco (2007), however.

The Recursive (Saddle-Point) Formulation

A solution to this model involves minimizing the Lagrangian with respect to γ and maximizing it with respect to $\boldsymbol{\mathcal{C}}$. This min-max problem is referred to as a saddle-point problem and can be formulated recursively like a standard Bellman equation.

Concretely, the value of a newly formed couple is

$$W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = \inf_{\{\gamma_{j,t}\geq 0\}_{j=1,t=0}^{2,T}} \sup_{\{\mathcal{C}_t\}_{t=0}^{T}} \mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0)$$

where, due to the law of iterated expectations, the Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_0 = & \sum_{j=1}^{2} \left\{ M_{j,0} U_j(\mathcal{C}_0, \mathcal{S}_0) - \gamma_{j,0} V_{j,0}^{m\to s}(\mathcal{S}_0) \right\} \\ & + \beta \mathbb{E}_0 \underbrace{\left[\mathbb{E}_1 \left[\sum_{t=0}^{T-1} \beta^t \sum_{j=1}^{2} \left\{ M_{j,t+1} U_j(\mathcal{C}_{t+1}, \mathcal{S}_{t+1}) - \gamma_{j,t+1} V_{j,t+1}^{m\to s}(\mathcal{S}_{t+1}) \right\} \right] \right]}_{=\mathcal{L}_1} \end{aligned}$$

with a slight abuse of notation. Denoting $W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = \mathcal{L}_0(\boldsymbol{\mathcal{C}}^\star, \boldsymbol{\gamma}^\star, \boldsymbol{\lambda}, \mathcal{S}_0)$ as the value when optimal choices are inserted, the problem can be written as

$$\begin{aligned} W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = & \inf_{\{\gamma_{j,0}\geq 0\}_{j=1}^{2}} \sup_{\mathcal{C}_0} \sum_{j=1}^{2} \left\{ M_{j,0} U_j(\mathcal{C}_0, \mathcal{S}_0) - \gamma_{j,0} V_{j,0}^{m\to s}(\mathcal{S}_0) \right\} \\ & + \beta \mathbb{E}_0 [W_1(\mathcal{S}_1, M_{1,0}, M_{2,0})]. \end{aligned}$$

In turn, for an arbitrary period t we have the recursive formulation

$$\begin{aligned} W_t(\mathcal{S}_t, M_{1,t-1}, M_{2,t-1}) = & \inf_{\{\gamma_{j,t}\geq 0\}_{j=1}^{2}} \sup_{\mathcal{C}_t} \sum_{j=1}^{2} \left\{ (M_{j,t-1} + \gamma_{j,t}) U_j(\mathcal{C}_t, \mathcal{S}_t) - \gamma_{j,t} V_{j,t}^{m\to s}(\mathcal{S}_t) \right\} \\ & + \beta \mathbb{E}_t [W_{t+1}(\mathcal{S}_{t+1}, M_{1,t}, M_{2,t})] \end{aligned} \tag{A.4}$$

where $M_{j,t}$ transitions as in eq. (A.3) and subject to state-transitions and other constraints.

A useful fact is that the individual weights can be scaled in any way (Marcet and Marimon, 2019). A convenient normalization, which we will use below, is

$$\mu_{t-1} = \frac{M_{1,t-1}}{M_{1,t-1} + M_{2,t-1}}$$

since we then only need to know one co-state, μ_{t-1}, rather than the two $M_{1,-1}, M_{2,-1}$.

Increasing μ_t then corresponds to increasing the shadow price on the participation constraint of member one, $\gamma_{1,t}$. In turn, all relevant state-variables of a couple are $\mathcal{S}_t$ and μ_{t-1}, optimal endogenous choices are $\mathcal{C}_t^\star(\mathcal{S}_t, \mu_{t-1})$ and $\mu_t^\star(\mathcal{S}_t, \mu_{t-1})$ and states transition according to

$$\begin{aligned} \mathcal{S}_{t+1} &\sim \Gamma(\mathcal{S}_t, \mathcal{C}_t) \\ \mu_t &= \mu_t^\star(\mathcal{S}_t, \mu_{t-1}). \end{aligned}$$

The updated bargaining weight, $\mu_t^\star(\mathcal{S}_t, \mu_{t-1})$, is a result of the intra-household bargaining process and is discussed in detail in the main text.

In essence, we solve the saddle-point problem in two steps. First, we check the corner solution, $\gamma_{1,t} = \gamma_{2,t} = 0$ and maximize over $\mathcal{C}_t$. We then check whether the forward-looking participation constraints are satisfied for both members. If they are, we are at the corner solution. If none of the participation constraints are satisfied, the couple divorces and if only one of the participation constraints are violated, of say member j, we find the lowest value of $\gamma_{j,t}$ (and associated optimal $\mathcal{C}_t$) that satisfies the participation constraint.