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Modelling Commodity Demands and Labour Supply with M-Demands

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Abstract

In the empirical modelling of demands and labour supply we often lack data on a full set of goods. The usual response is to invoke separability assumptions. Here we present an alternative based on modelling demands as a function of prices and the quantity of a reference good rather than total expenditure. We term such demands m-demands. The advantage of this approach is that we make maximum use of the data to hand without invoking implausible separability assumptions. In the theory section quasi-Slutsky conditions are derived and some structural and separability conditions are presented. We also derive functional forms for empirical work. Finally an empirical illustraion on Canadian expenditure data is given. This illustrates both the implementation of the ideas presented in the theory section and the empirical costs of not having a full set of data.

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1. Introduction.

The empirical structural modelling of commodity demands and labour supply requires the specification of these as functions of demographics, prices and wages and some measure of income or wealth. There are two principal candidates for the latter. One choice is total expenditure (for demand equations or 'total expenditure minus earnings' for joint labour supply and commodity demand systems) to give Marshallian (or uncompensated) demands. Often, however, we do not observe all the goods that the agent buys so that we cannot construct a measure of total expenditure. For example, in a neo-classical model of non-durables, services and durables we need to observe the stock of durables. Alternatively, we can condition on the (unobservable) marginal utility of money (which is a monotone function of currently perceived lifetime wealth) to give Frisch demands (see, for example, Heckman (1974a), MaCurdy (1981) or Browning, Deaton and Irish (1985)). This approach has the problem that we need panel data on consumption to deal with the unobservability of the marginal utility of expenditure. In this paper we discuss an alternative to these two demands which is designed to maximise the preference information we can recover from an incomplete set of expenditure information. This is to model a sub-set of goods and to condition on the level of another (reference) good rather than total expenditure or the marginal utility of money. If the reference good is normal then it is a satisfactory measure of utility (conditional on prices). These demands can be derived from marginal rates of substitution so we term them *m-demands.* In this paper we present the theoretical underpinnings for m-demands. These demands have been used (either explicitly or implicitly) in Heckman (1974b), Altonji (1986), Meghir and Weber (1996) and Attanasio and MaCurdy (1997). As we shall discuss below, m-demands are also implicitly used in calibration exercises for dynamic general equilibrium models.

An agent buys a set of goods[1] $(q_1, q_2, ...q_n, z)$with associated market prices $(p_1, p_2, ...p_n, r) = (\mathbf{p}, r)$. The investigator observes all these prices but only observes a subset of goods $(q_1, q_2, ...q_m, z)$ where m is typically smaller than n. Then we model the demands for all but one of the observed goods $(q_1, q_2, ...q_m)$ as functions of all prices and the level of one observed good, z, which we shall refer to as the *reference good.* Thus we model $q_i = f^i(p_1, p_2, ...p_n, r, z)$ and refer to these functions as m-demands. Of course, in empirical work we shall have to take account of the possible endogeneity of the level of the reference good but this raises no new

[1] In all that follows we consider only commodity demands; the extension to labour supply is conceptually trivial (if somewhat more awkward empirically).

issues since in Marshallian demands we have to allow for the endogeneity of total expenditure and in Frisch demands for the unobservability of the marginal utility of money. Note that m-demands are very different to the conditional Marshallian demands of Pollak (1976) and Browning and Meghir (1991) which take demands for non-reference goods as functions of the prices of the non-reference goods, total expenditure on these goods and a conditioning good z; that is, conditional Marshallian demands take the form $q_i(p_1, p_2, ...p_n, x, z)$ where $x = \sum_{i=1}^{n} p_i q_i$.

There are a number of advantages to modelling m-demands. First there is no need to observe all quantities to model the demands for a subset of goods. If we go the Marshallian route then we usually need to assume some (implausible) separability to estimate demands in such a situation. We present three examples. First, consider the PSID in which we observe two food measures ('food at home' and 'food in restaurants') and two labour supplies (for married couples). Using an m-demand approach, we can model the two labour supplies and food in restaurants as functions of wages, the prices of all goods (which are publicly available even for those goods for which we do not observe quantities in the PSID) and the quantity of food at home. The latter is certainly normal in any measure of wealth or income we might consider so it is a prime candidate as a reference good. As a second example, note that in expenditure surveys we do not usually observe the stock of durables. Suppose we want to model non-durables in a Marshallian system and not assume that they are separable from durables. If we adopt a user-cost specification, we need to include the value of the stock of durables (using the user cost as price) in total expenditure. It is precisely because we do not observe stocks of durables that this avenue is not available. In an m-demand approach we can model non-durables and condition on the prices of prices, the user cost of durables (which can be constructed from observable prices and interest rates) and the level of some good that all households purchase (for example, 'food at home'). The final example is also the most widely used: in empirical work on intertemporal allocation we never observe total lifetime expenditure by any person or household. Instead, we have to consider sub-periods of the lifetime. One way to proceed is to condition on last period's consumption (the so-called 'Euler equation' approach). That is, assuming intertemporal additivity we model current consumption as a function of current and lagged (discounted) prices (or the first difference of the log of them, which equals the real interest rates) and lagged consumption. Thus lagged consumption is the reference good in the m-demand for current consumption.

A second advantage is that, like Marshallian demands, the modelling of within period m-demands is robust to some controversial assumptions often made for intertemporal allocation. For example, even if agents are liquidity constrained the m-demands are correctly specified.

The third advantage stems from our interest in estimating the parameters of intertemporal allocation. For example, from a set of Frisch (or λ-constant) demands and labour supply we can recover everything we need to know about preferences for policy analysis and dynamic general equilibrium (DGE) modelling. The problem with estimating such a system is that we have no long panel that contains all the requisite consumption information. The m-demand approach provides a coherent framework for using consumption and labour supply information from different data sources. From m-demands estimated on expenditure surveys (which include labour supply information) we can recover almost all of the parameters of interest; the only missing element are the parameters for the intertemporal allocation of the reference good. As it happens we do have a long panel (the PSID) that contains enough information to estimate these remaining parameters *without invoking within period separability assumptions.* Thus the use of m-demands on cross-section expenditure surveys containing information on all goods and a long panel containing information on the reference good allows us to recover the full set of intertemporal parameters. Thus we could estimate the intertemporal substitution elasticities for both consumption and labour supply which both have potentially important implications for many policy debates.

A fourth advantage is that in empirical work it turns out to be easier to incorporate heterogeneity in preferences in m-demands than it is for Marshallian demands. Finally, certain theoretical restrictions are more easily specified and imposed for m-demands than for other demands. We shall present an example taken from the dynamic general equilibrium literature below when we have developed the theory some more.

There is also one drawback to the use of m-demands, namely that if we have incomplete expenditure data we cannot recover all of the Marshallian demands. Nor can we recover the generating preferences even if the quasi-Slutsky conditions derived below are imposed. This is simply a converse of the fact that we need less information to estimate m-demands. Indeed, m-demands can be seen as a way of recovering the maximum amount of information regarding preferences given that we do not have full information on quantities. Below we shall also discuss how we can incorporate additional information on preferences. At present, it is an open question as to exactly how much we can recover non-parametrically of preferences from modelling a subset of m-demands.

2. Deriving m-demands.

There are a number of routes to the derivation of m-demands. The first of these is to simply write down a system of m-demand functions which are convenient to estimate and then to impose the quasi-Slutsky conditions given in the next section. As we shall see below, this is a 'back and forth' process since the first demands we write down usually have very restrictive forms when the quasi-Slutsky conditions are imposed and we have to go back and modify the unrestricted form.

A second approach is to take some widely used Marshallian demand system and then to invert it on the reference good and substitute this into the other equations. Suppose we have the Marshallian system:

$$\begin{aligned} q_i &= g^i(\mathbf{p}, r, x), \qquad i = 1...n \\ z &= g^z(\mathbf{p}, r, x) \end{aligned} \tag{2.1}$$

where x is total expenditure on all $(n+1)$ goods. Since the reference good is normal we can invert the final equation to give $x = \theta(\mathbf{p}, r, z)$ and substitute to give:

$$q_i = g^i(\mathbf{p}, r, \theta(\mathbf{p}, r, z)) = f^i(\mathbf{p}, r, z) \tag{2.2}$$

As an example that we shall carry through this section, consider the three good linear expenditure system (LES)[2] derived from the utility function:

$$u(q_1, q_2, z) = \alpha * ln(q_1 - a) + \beta * ln(q_2 - b) + (1 - \alpha - \beta) * ln(z - d) \tag{2.3}$$

This gives the following Marshallian demand for good z:

$$z = d + \frac{(1 - \alpha - \beta)(x - ap_1 - bp_2 - dr)}{r} \tag{2.4}$$

Inverting on x gives:

$$\theta(p_1, p_2, r, z) = ap_1 + bp_2 + rd + \frac{r(z - d)}{(1 - \alpha - \beta)} \tag{2.5}$$

[2] This form is not very useful for empirical work. We shall suggest better below, but for now the LES illustrates many of the issues in a simple way. It also suggests some structural and separability arguments since it represents preferences that are additive and quasi-homothetic. We shall also return to this below.

and substituting into the LES Marshallian demands gives the m-demand:

$$
\begin{aligned}
q_1 &= a + \frac{\alpha}{(1-\alpha-\beta)} \frac{r}{p_1}(z-d) \\
q_2 &= b + \frac{\beta}{(1-\alpha-\beta)} \frac{r}{p_2}(z-d)
\end{aligned}
\tag{2.6}
$$

so that for the LES, m-demands are an affine function of the level of the reference good and are independent of the price of other non-reference goods. Although this approach is attractively simple, it has the problem that we can only find closed form representations for $\theta(.)$ for a very restricted set of demand systems. As an example, the widely used Almost Ideal system does not lend itself to this approach.

The third approach begins with the direct representation $\upsilon(\mathbf{q}, z)$ and uses the first order conditions for the marginal rates of substitution (hence the name *m-demands*). The first order conditions for maximisation are:

$$
\frac{\upsilon_i(\mathbf{q}, z)}{\upsilon_z(\mathbf{q}, z)} = \frac{p_i}{r}, \qquad i = 1, 2...n \tag{2.7}
$$

If we solve these n equations for the quantities of the non-reference goods[3] then we have m-demands $q_i(\mathbf{p}, r, z)$ which are zero homogeneous in $(\mathbf{p}, r)$. For example, for the LES example we have:

$$
\begin{aligned}
\frac{\alpha(z-d)}{(q_1-a)(1-\alpha-\beta)} &= \frac{p_1}{r} \\
\frac{\beta(z-d)}{(q_2-b)(1-\alpha-\beta)} &= \frac{p_2}{r}
\end{aligned}
\tag{2.8}
$$

Solving these for (q_1, q_2) gives the m-demand in equation in (2.6). This approach is somewhat restrictive since it is only for particular forms of the utility function (such as the LES) that we can find closed forms for the demands. On the other hand, the fact that we do not simultaneously have to solve for the budget constraint (which has a different from to the mrs equations) does make this route more widely applicable than it is for Marshallian demands.

The traditional way to derive demands is to have recourse to 'dual' approaches. Given a cost function representation for preferences $c(\mathbf{p}, r, u)$ we can derive Hicksian demands $q_i(\mathbf{p}, r, u)$ $(= c_i)$ and $z(\mathbf{p}, r, u)$ $(= c_r)$. If the reference good z is a

[3]In the m-demand approach we have to solve for all equations, even for goods for which we do not observe the quantities.

normal good for all $(\mathbf{p}, r, u)$ then it is strictly increasing in the utility level u so that we can invert the Hicksian demand for the reference good to give:

$$z = c_r(\mathbf{p}, r, u) \Rightarrow u = c_r^{-1}(\mathbf{p}, r, z) = \psi(\mathbf{p}, r, z) \tag{2.9}$$

Although the function $\psi(\mathbf{p}, r, z)$ has some of the properties of a preference representation it is *not* a valid representation in the sense that there is a one-one mapping from preferences to these functions. For example, if we replace $c(\mathbf{p}, r, u)$ with $c(\mathbf{p}, r, u) + \lambda(\mathbf{p}, u)$ (which represents different preferences) we obtain the same $\psi(\mathbf{p}, r, z)$function. If we substitute for u into the other Hicksian demands we have the m-demands:

$$q_i(\mathbf{p}, r, \psi(\mathbf{p}, r, z)) = q_i(\mathbf{p}, r, z) \tag{2.10}$$

This procedure is very close to the usual procedure for deriving Marshallian demands from the cost function except that we substitute in the z-conditional function $\psi(\mathbf{p}, r, z)$ rather than the indirect utility function $V(\mathbf{p}, r, x)$.

To illustrate with the LES example, note that the cost function is given by:

$$c(\mathbf{p}, r, u) = ap_1 + bp_2 + dr + \left(\alpha^{-\alpha}\beta^{-\beta}(1-\alpha-\beta)^{-(1-\alpha-\beta)}\right)\left(p_1^{\alpha}p_2^{\beta}r^{(1-\alpha-\beta)}\right) exp(u) \tag{2.11}$$

so that:

$$\psi(\mathbf{p}, r, z) = k - \alpha \ln p_1 - \beta \ln p_2 + (\alpha+\beta)\ln r + \ln(z-d)$$

Substituting this into the Hicksian demands for the other goods once again gives the m-demands presented in equation 2.6 above.

Thus we have various ways to derive m-demands; Murphy (1998) extends this list to provide conditional cost and profit function derivations. Which route to m-demands is most useful will in general depend on the context and the facility of the researcher in handling different representations. To finish this section, we present another form of m-demands that is more useful in empirical work. When modelling Marshallian demands we almost always model budget shares rather than demands or expenditures. In the context taken here we cannot do this since we are assuming that total expenditure (the denominator for budget shares) is not observed. We can, however, define *relative shares* which give the expenditures on the goods of interest relative to the expenditure on the reference good :

$$\omega_i = \frac{p_i q_i}{rz} = \frac{p_i f^i(\mathbf{p}, r, z)}{rz} = h^i(\ln \mathbf{p}, \ln r, \ln z) \tag{2.12}$$

where we take the function to be defined on log prices and the log of the reference good, since this is usually more convenient for relative shares. For completeness, we present the relative shares for the LES system even though this does not take a natural relative share form:

$$\omega_1 = \frac{ap_1}{rz} + \frac{\alpha}{(1-\alpha-\beta)}(1-\frac{d}{z}) \tag{2.13}$$

For later use, it is useful to note how the derivatives of the demands and the relative shares are related:

$$\begin{aligned}
\frac{\partial q_i}{\partial p_i} &= \left(\frac{\partial \omega_i}{\partial \ln p_i} - \omega_i\right)\frac{rz}{p_i^2} \\
\frac{\partial q_i}{\partial p_j} &= \frac{\partial \omega_i}{\partial \ln p_j}\frac{rz}{p_i p_j} \qquad i \neq j \\
\frac{\partial q_i}{\partial r} &= \left(\frac{\partial \omega_i}{\partial \ln r} + \omega_i\right)\frac{z}{p_i} \\
\frac{\partial q_i}{\partial z} &= \left(\frac{\partial \omega_i}{\partial \ln z} + \omega_i\right)\frac{r}{p_i}
\end{aligned} \tag{2.14}$$

We turn now to the derivation of the Slutsky type restrictions that integrability imposes on m-demands.

3. Theory.

3.1. Slutsky conditions.

In this section we derive the Slutsky conditions on m-demands. Given the m-demands $q_i = f^i(\mathbf{p}, r, z)$ or the relative share functions $\omega_i = h^i(\ln \mathbf{p}, \ln r, z)$ we have the following restrictions[4].

Theorem 3.1. *The m-demands satisfy the following:*

- *Each $q_i = f^i(\mathbf{p}, r, z)$ and $\omega_i = h^i(\ln \mathbf{p}, \ln r, \ln z)$ is zero homogeneous in $(\mathbf{p}, r)$.*

[4] We have here assumed differentiability whenever we need it. This could be avoided by the use of sub-differentials and much more notation.

- *The matrices:*

$$S = \left[\frac{\partial q_i}{\partial p_j} + \frac{\partial q_i}{\partial z}\frac{\partial q_j}{\partial r}\right]_{i,j} \qquad i,j = 1,2,...n$$

and

$$W = \left[\frac{\partial \omega_i}{\partial \ln p_j} + \frac{\partial \omega_i}{\partial \ln z}\frac{\partial \omega_j}{\partial \ln r} + \omega_i \frac{\partial \omega_j}{\partial \ln r} + \omega_j \frac{\partial \omega_i}{\partial \ln z}\right]_{i,j}$$

are symmetric.

Proof. The homogeneity conditions hold trivially from the marginal rate of substitution derivation of the m-demands.

To derive the symmetry condition for the levels of m-demands, take the cost function $c(\mathbf{p}, r, u)$ and the notation of the previous section. We have the following identity:

$$z \equiv c_r(\mathbf{p}, r, \psi(\mathbf{p}, r, z))$$

Since this holds identically, we can take derivatives with respect to z, p_i and r:

$$\begin{aligned} c_{ru}\psi_z &= 1 \\ c_{rr} + c_{ru}\psi_r &= 0 \\ c_{ri} + c_{ru}\psi_i &= 0 \end{aligned} \tag{3.1}$$

The m-demand for good i is given by:

$$q_i = c_i(\mathbf{p}, r, \psi(\mathbf{p}, r, z))$$

Taking derivatives and substituting from 3.1 we have:

$$\begin{aligned} \frac{\partial q_i}{\partial z} &= c_{iu}\psi_z = \frac{c_{iu}}{c_{ru}} \\ \frac{\partial q_i}{\partial r} &= c_{ir} + c_{iu}\psi_r = c_{ir} - \frac{c_{iu}}{c_{ru}}c_{rr} \\ \frac{\partial q_i}{\partial p_j} &= c_{ij} + c_{iu}\psi_j = c_{ij} - \frac{c_{iu}}{c_{ru}}c_{rj} \end{aligned} \tag{3.2}$$

Solving for these we have:

$$\frac{\partial q_i}{\partial p_j} = c_{ij} - \frac{\partial q_i}{\partial z}\left(\frac{\partial q_j}{\partial r} + \frac{\partial q_j}{\partial z}c_{rr}\right) \tag{3.3}$$

Re-arranging we have:

$$\frac{\partial q_i}{\partial p_j} + \frac{\partial q_i}{\partial z}\frac{\partial q_j}{\partial r} = c_{ij} - \frac{\partial q_i}{\partial z}\frac{\partial q_j}{\partial z}c_{rr} \tag{3.4}$$

Since the right hand side is symmetric in (i, j) so is the left hand side. Thus the condition in the statement of the theorem holds.

To derive the relative share restrictions, take the levels condition and substitute in the relationships given in (2.14) ■

This shows that there are homogeneity and symmetry restrictions for m-demands. The symmetry conditions derived above are reminiscent of the usual Slutsky conditions except that we replace x with z and q_j with $\partial q_j/\partial r$. Note that if we model a subset of the non-reference goods then the symmetry conditions only apply to the goods we model (that is, the cross price term with respect to the price of a good that is not observed is unrestricted) . There are no adding-up restrictions since we do not model the reference good. Murphy (1998) uses a conditional cost function argument to show that m-demand systems must also satisfy a negativity condition. Specifically, he shows that the matrix:

$$\left[\frac{\partial q_i}{\partial p_j} - \frac{\partial q_i}{\partial r}\frac{\partial q_j}{\partial z}\right]_{i,j} \qquad i, j = 1, 2, ...n$$

is symmetric and negative semi-definite. The symmetry condition is the same as the one given in the Theorem above. The negativity result implies that own price responses of the following form:

$$\frac{\partial q_i}{\partial p_i} - \frac{\partial q_i}{\partial r}\frac{\partial q_i}{\partial z}$$

are non-positive.

3.2. Some applications

Any set of m-demands must satisfy the quasi-Slutsky conditions above if they are to be integrable. Note, however, that some authors consider systems with only two good (usually in the context of labour supply in which preferences are defined over one labour supply and income). In this case the symmetry conditions are nugatory and we have only the homogeneity and own price negativity restrictions. A leading example of this is in calibration exercises for dynamic general equilibrium models which take a one good, one labour supply model with a representative agent (see, for example, Hansen (1996)). Let q be leisure with p as the nominal wage rate and z be consumption with price r, so that $p/r = w$ is the real wage. Then we have the m-demand function for leisure:

$$q = f(\frac{p}{r}, z) = f(w, z) \tag{3.5}$$

In the DGE literature it is common to use two long run averages and one cross-section fact to derive both the functional form and the parameters for the preferences generating this demand function. In broad terms, the two long run facts are that over the post-war period consumption growth and real wage growth are equal and positive and labour supply has remained constant[5]. That is:

$$d\ln z = d\ln w > 0 \text{ and } d\ln q = 0 \tag{3.6}$$

From equation 3.5 this implies that the m-demand for leisure is zero homogeneous in (w, z):

$$wf_w + zf_z = 0 \Rightarrow q = g(\frac{w}{z}) \tag{3.7}$$

Using this and the first order conditions, we have:

$$\frac{u_q(q, z)}{u_z(q, z)} = w = zg^{-1}(q) \tag{3.8}$$

The general solution to this partial differential equation is:

$$u(q, z) = F(z\exp(\int g^{-1}(q)dq)) \tag{3.9}$$

Particular solutions are then usually found by imposing functional form restrictions and using cross-section information on the proportion of time spent working.

We now illustrate the use of the quasi-Slutsky conditions by considering three sets of preferences; for simplicity we consider only three good systems (that is, the two goods to be modelled and the reference good). First, take the LES m-demands given in equation 2.6. The levels cross terms are given by:

$$s_{12} = \frac{\partial q_1}{\partial p_2} + \frac{\partial q_1}{\partial z}\frac{\partial q_2}{\partial r} = 0 + \left(\frac{\alpha}{1-\alpha-\beta}\frac{r}{p_1}\right)\left(\frac{\beta}{1-\alpha-\beta}\frac{(z-d)}{p_2}\right)$$

[5] A more sophisticated approach uses the long run averages of these growth rates, but the derivations are similar.

which is symmetric.

The next example derives from the observation that since we do not have any adding-up restriction the double-log form is an attractive suggestion (see Attanasio and MaCurdy (1996)). Thus we have (for the two good example) the homogeneous form:

$$\ln q_i = \alpha_i + \gamma_{i1}(\ln p_1 - \ln r) + \gamma_{i2}(\ln p_2 - \ln r) + \beta_i \ln z \quad i = 1, 2 \tag{3.10}$$

Ignoring solutions for the quasi-Slutsky restrictions that restrict the β_i's we have the following solution:

$$\gamma_{12} = 0, \gamma_{21} = 0 \text{ and } \frac{\gamma_{22}}{\gamma_{11}} = \frac{\beta_2}{\beta_1}$$

This restricts the cross price terms to be zero and the own price terms to be closely related by the coefficients on the reference good. Thus the promising double-log form turns out to be very restrictive if we wish to impose the integrability conditions. We can also consider a relative share system that is linear in logs. In homogeneous form this is:

$$w_i = \alpha_i + \gamma_{i1}(\ln p_1 - \ln r) + \gamma_{i2}(\ln p_2 - \ln r) + \beta_i \ln z \quad i = 1, 2 \tag{3.11}$$

There are two sets of solutions for the quasi-Slutsky restrictions. The first set is very unattractive since it has the slope coefficients in the second equation being proportional to the slope coefficients in the first equation and γ_{11} as a (complicated) function of the other parameters, including the constant terms. The other set of restrictions are:

$$\gamma_{21} = \gamma_{12} \text{ and } \gamma_{i1} + \gamma_{i2} + \beta_i = 0 \quad i = 1, 2 \tag{3.12}$$

In this form we have symmetric cross-price terms and the slope coefficients within each equation adding to zero.

The final example we consider is also the most important - the use of m-demands in the analysis of intertemporal allocation. Here we present an illustration with consumption (c_t in period t) and labour supply (h_t). Let Frisch demands and supply be given by:

$$\begin{aligned} c_t &= f(p_t, w_t, \lambda_t) \\ h_t &= g(p_t, w_t, \lambda_t) \end{aligned} \tag{3.13}$$

where w_t is the discounted wage in period t, p_t is the discounted price of consumption and λ_t is the marginal utility of (discounted) expenditure in period t. If we invert the consumption relationship on λ_t to give $\lambda_t = k(p_t, w_t, c_t)$ and substitute in the labour supply equation then we have the m-demand:

$$h_t = g(p_t, w_t, k(p_t, w_t, c_t)) = h(p_t, w_t, c_t) \tag{3.14}$$

The parameters of $h(.)$ can be identified from cross-section variation in real wages and consumption. However, this does not generally suffice to identify all of the parameters of $f(.)$ and $g(.)$ in (3.13). To do this we need intertemporal variation in prices and wages. Using the conventional Euler equation:

$$\lambda_{t+1} = \lambda_t + \varepsilon_{t+1} \tag{3.15}$$

where ε_{t+1} is a 'surprise' term and substituting, we have:

$$c_{t+1} = f(p_{t+1}, w_{t+1}, \lambda_{t+1}) = f(p_{t+1}, w_{t+1}, k(p_t, w_t, c_t) + \varepsilon_{t+1}) \tag{3.16}$$

This is a general formulation for the usual Euler equation that relates current consumption to lagged consumption and current and lagged wages and prices. We can identify all the parameters of $f(.)$ and $k(.)$ if we have variation in discounted prices and wages. With estimates of (3.14) this gives all the parameters of (3.13). This approach to intertemporal modelling facilitates the choice of functional form so that, for example, $f(.)$ is additive in λ or so that we can incorporate heterogeneity in a tractable yet non-trivial way; we shall return to the latter below.

4. Relationship with other demands.

How do the m-demands we have above relate to more conventional demands? To see the relationship to Marshallian (uncompensated) demands, denote the Marshallian demand function for good i by $g^i(\mathbf{p}, r, x)$ (and similarly for $z = g^z(\mathbf{p}, r, x)$) where x is total expenditure. Then we have:

$$q_i = f^i(\mathbf{p}, r, z) = f^i(\mathbf{p}, r, g^z(\mathbf{p}, r, x)) = g^i(\mathbf{p}, r, x) \tag{4.1}$$

Taking derivatives through this with respect to x we have:

$$\frac{\partial q_i}{\partial x} = \frac{\partial q_i}{\partial z}\frac{\partial z}{\partial x} \Rightarrow \frac{\partial \ln q_i}{\partial \ln x} = \frac{\partial \ln q_i}{\partial \ln z}\frac{\partial \ln z}{\partial \ln x} \tag{4.2}$$

From the estimation of m-demands we can recover the first term on the right hand side of this expression. If we observe a full set of quantities then we could calculate x and we could also estimate the second term but in this case it is easier to simply estimate the uncompensated demands (the left hand side) directly. Suppose that we don't have a full set of quantities. Specifically, consider the use of the PSID and the use of 'food at home' as the reference good for 'food in restaurants' and male and female labour supply. The missing information is then the relationship between expenditures on food at home and total expenditure. This, however, is one of the most thoroughly investigated empirical relationships in economics and given the wealth of information we have about the unobserved food income elasticity we can put fairly tight bounds on plausible values for this term and hence bound the left hand side as well. From equation (4.1) we have that good i is a luxury if:

$$\frac{\partial \ln q_i}{\partial \ln x} > 1 \Leftrightarrow \frac{\partial \ln q_i}{\partial \ln z} > \left(\frac{\partial \ln z}{\partial \ln x}\right)^{-1} \tag{4.3}$$

If we take the reference good to be 'food at home' then a reasonable value for the food elasticity is about 0.4 so that good i is a luxury if the elasticity with respect to the reference good is above 2.5.

Similar manipulations give the uncompensated price responses as:

$$\frac{\partial \ln q_i}{\partial \ln p_j}|_x = \frac{\partial \ln q_i}{\partial \ln p_j}|_z + \frac{\partial \ln q_i}{\partial \ln z}\frac{\partial \ln z}{\partial \ln p_j}|_x \tag{4.4}$$

and once again we may be able to obtain reliable information about bounds on the unobservable final term from other sources. From this equation we see that the own price m-demand response for good i is negative if it is a normal good (so that the left hand side is negative and the first term in the product on the right hand side is positive) and it is not a strong gross complement for the reference good (so that the final term in the product on the right hand side in not 'too negative').

Of more interest are the links to compensated or Hicksian responses since these are used in welfare calculations. For example, the compensating variation for a change δ in the price of good i is given by:

$$CV = c(p_1, ...p_i + \delta, ...p_n, r, u) - c(p_1, ...p_i, ...p_n, r, u)$$

which has the convenient second order approximation:

$$CV = \Delta c(\mathbf{p}, r, u) \simeq c_i(\mathbf{p}, r, u)\Delta p_i + \frac{1}{2}c_{ii}(\mathbf{p}, r, u)(\Delta p_i)\hat{}2 \tag{4.5}$$

$$= \quad q_i \Delta p_i + \frac{1}{2} \frac{\partial q_i}{\partial p_i}|_u (\Delta p_i)\hat{}2$$

so that knowing the levels of demands and own price compensated responses gives an approximation to the compensating variation. To see how much of this we can recover from m-demands, first note that the relationship between m-demands and compensated demands is given by:

$$q_i(\mathbf{p}, r, u) = q_i(\mathbf{p}, r, c_r(\mathbf{p}, r, u)) \tag{4.6}$$

where $c_r(.)$ is the partial of the cost function with respect to the price of the reference good. Differentiating with respect to the price of i we have:

$$\frac{\partial q_i}{\partial p_i}|_u = \frac{\partial q_i}{\partial p_i}|_z + \frac{\partial q_i}{\partial z} c_{ri} = \frac{\partial q_i}{\partial p_i}|_z + \frac{\partial q_i}{\partial z} \frac{\partial q_i}{\partial r}|_u \tag{4.7}$$

Thus the m-demand own price response and compensated price response for good i coincide if and only if this good is Hicks independent of the reference good $(\frac{\partial q_i}{\partial r}|_u = 0)$. This will be a reasonable approximation for some of the broad commodity groups we usually model. For example, it may not be too unreasonable to ignore the compensated cross price terms between, say, food at home (our canonical reference good) and clothing. On the other hand, there are some broad commodities for which this will not be the case. An obvious example is restaurant expenditures which are surely a substitute for food at home. In this case using the m-demand price response in (4.5) rather than the compensated response will lead to an under-estimate of the welfare cost of a rise in the price of restaurant food. This is because the m-demand response does not allow for the possibility of substituting food at home for restaurant expenditures. Once again, it may be possible to use supplementary information on compensated price responses to provide bounds on the second order approximation welfare effect.

We can also perform similar manipulations to derive Frisch (or $\lambda-$constant) responses. Let the Frisch demand for the reference good be given by:

$$z = \chi(\mathbf{p}, r, \lambda) \tag{4.8}$$

where λ is the marginal utility of money. Then the Frisch demand for good i is given by $h^i(\mathbf{p}, r, \lambda) = f^i(\mathbf{p}, r, \chi(\mathbf{p}, r, \lambda))$. Thus the Frisch price responses are:

$$\frac{\partial q_i}{\partial p_j}|_\lambda = \frac{\partial q_i}{\partial p_j}|_z + \frac{\partial q_i}{\partial z} \frac{\partial z}{\partial p_j}|_\lambda \tag{4.9}$$

Once again, we have an unobservable term on the right hand side (the final derivative) but now much less is known about it than is known about the previous unobservable (the uncompensated income elasticity for food at home). If we are willing to assume that good i is additively separable from the reference good then $\frac{\partial z}{\partial p_j}|_\lambda = 0$ and the m-demand response is the Frisch response. Generally, however, this will not be a reasonable assumption and we need to take account of Frisch cross effects. However, if we consider again the use of the PSID and 'food at home' as the reference good then we are in an even better position than for the Marshallian response since the PSID provides a panel of observations on 'food at home' and labour supplies. Thus we can estimate the Frisch response for 'food at home' even if we don't observe the quantities of other goods. This can be done *without supplementary (and implausible) separability assumptions* since we need only observe prices, which are publicly available, to estimate $z(\mathbf{p}, r, \lambda)$. The latter can be done using conventional Euler equation methods.

5. Structure and separability.

In this section we consider the effects of different separability structures on the form of m-demands. Even if we do not observe a full set of goods, it is still possible to test for various separability structures. In the following we consider n goods and a reference good with only m of the non-reference goods being observable.

The first structural argument is important for the analysis of intertemporal allocation where we often assume that preferences are additive over time.

Proposition 5.1. *If preferences are additive over goods:*

$$u(q_1, ...q_n, z) = \sum_i \phi_i(q_i) + \phi_z(z)$$

then the m-demand for good i is independent of other non-reference good prices:

$$q_i = f^i\left(\frac{p_i}{r}, z\right)$$

Moreover, own price responses are negative.

When modelling intertemporal allocation we take the reference good to be last period's consumption so that this period's consumption depends only on last period's consumption and the relative price of the two consumptions; that is, the real interest rate.

The next two propositions are useful in the context of testing for weak separability.

Proposition 5.2. *If preferences for goods* $(q_1, ...q_{n-1}, z)$ *are separable from good* n*:*

$$u(q_0, q_1, ...q_n, z) = V(q_n, \phi(q_1, ...q_{n-1}, z))$$

then the m-demands for good $i \neq n$ *are independent of the level and price of good* n*:*

$$q_i = f^i(p_1, ...p_{n-1}, r, z)$$

Thus we can easily test for separability in the m-demand context by testing for exclusion restrictions. This is very similar to the conditional demand tests for separability of Browning and Meghir (1991) except that there quantities are excluded and here we exclude prices. We also have:

Proposition 5.3. *If preferences over the goods* $(q_1, ...q_n)$ *are separable from the level of the reference good:*

$$u(q_1, ...q_n, z) = V(z, \phi(q_1, ...q_n))$$

then m-demands are independent of the price of the reference good:

$$q_i = f^i(p_1, ...p_n, z)$$

This provides another simple test for separability.

The next proposition deals with another facet of structure.

Proposition 5.4. *If preferences are quasi-homothetic:*

$$c(p_1, ...p_n, r, u) = a(p_1, ...p_n, r) + b(p_1, ...p_n, r)u$$

then m-demands are affine in z*:*

$$q_i = \alpha^i(p_1, ...p_n, r) + \beta^i(p_1, ...p_n, r)z$$

A closely related result which follows immediately is that if preferences are homothetic then m-demands are linear in z (that is the α^i's in the proposition are zero). Note that the converse does not hold; this is a consequence of the fact that we do not observe the full set of quantities.

6. An empirical application.

6.1. Functional form.

We now present an empirical application using Canadian household expenditure data. As emphasised in the introduction it is our belief that the principal context for m-demands is in modelling jointly labour supply and demands using complementary data sets such as the PSID and the U.S. CEX but the empirical illustration presented here brings out many of the important modelling aspects. As discussed above, when we estimate Marshallian demand systems we usually model budget shares; here we shall model relative shares (the ratio of expenditure on the good to be modelled to expenditure on the reference good). To motivate the system, consider first the modelling of within period demands (with prices set to unity) and let the relative share be a quadratic in the log of the reference good:

$$w_i = \frac{q_i}{z} = \alpha_i(\mathbf{a}) + \beta_i \ln(z) + \delta_i \ln(z)^2 \tag{6.1}$$

where $\mathbf{a}$ is a vector of demographics. This is flexible and very easy to estimate if we assume, for example, that demographics enter linearly:

$$\alpha_i(\mathbf{a}) = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} a_k \tag{6.2}$$

This form also admits of straightforward interpretations. Some differentiation gives that the Marshallian income elasticity is given by:

$$\frac{\partial \ln q_i}{\partial \ln x} = \left(1 + \frac{\beta_i + 2\delta_i \ln(z)}{w_i}\right) \frac{\partial \ln z}{\partial \ln x} \tag{6.3}$$

Thus the Marshallian income elasticity of the good being modelled is greater than the reference good income elasticity if and only if $\beta_i + 2\delta_i \ln(z) > 0$. Finally, we shall show below that the quadratic form seems to capture adequately non-linearities in the relationship between the non-reference goods and the reference good.

To incorporate prices we adopt a form that generalises equation (6.1). There are many such forms but considerable experimentation lead to the following:

$$w_i = \frac{p_i q_i}{rz} = \alpha_i(\mathbf{a}) + \sum_{j=1}^{n} \gamma_{ij} \ln(p_j) + \beta_i(\ln(rz) - \ln(p_i)) + \delta_i(\ln(rz) - \ln(p_i))^2 \tag{6.4}$$

This functional form maintains the flexibility of the within period form whilst allowing for price responses in an additive way. The system is easy to estimate since it is linear in parameters but note that the right hand side varies from equation to equation so that we have to use systems estimation techniques in estimation. Finally, the quasi-Slutsky restrictions are simple to impose (not surprisingly, since this was the criterion used to select a generalisation of the within period quadratic form). They are:

$$\begin{aligned} \text{Homogeneity} &: \quad \sum_{j=1}^{n} \gamma_{ij} = 0 \quad \text{for all } i \\ \text{Symmetry} &: \quad \gamma_{ij} = \gamma_{ji} \quad \text{for all } i, j \text{ for which the good is observed} \end{aligned} \tag{6.5}$$

This system is reminiscent of the quadratic AI Marshallian system (QAIDS) of Banks, Blundell and Lewbel (1997) but it represents different preferences. It is also a good deal easier to estimate since integrability in the QAIDS requires that both the deflator for total expenditure and the coefficients on the quadratic term be functions of prices, which is not required here.

6.2. Heterogeneity.

In general we must allow for the fact that different people will have different tastes. There are two broad approaches to introducing heterogeneity into demand systems. The first is to allow for heterogeneity in a general way that does not depend on the particular functional form and the second is to allow for it in a way that is convenient for estimation of a particular functional form. Examples of the general approach are given by Heckman (1974b) and Brown and Matzkin (1997) who respectively allow for multiplicative and additive effects on the marginal rates of substitution. For example, for the multiplicative form we have that the mrs for household h is given by:

$$\frac{\upsilon_i^h(\mathbf{q}, z)}{\upsilon_z^h(\mathbf{q}, z)} = \frac{\upsilon_i(\mathbf{q}, z)}{\upsilon_z(\mathbf{q}, z)} \exp(-\varepsilon_{ih}) \tag{6.6}$$

where $\upsilon(\mathbf{q}, z)$ is the common utility function and ε_{ih} is a household taste shifter for good i. Since the household specific mrs's are then equated to relative prices this implicitly amounts to assuming that heterogeneity only enters as though households face different prices for the non-reference goods so that:

$$\frac{\upsilon_i(\mathbf{q}, z)}{\upsilon_z(\mathbf{q}, z)} = \frac{p_i \exp(\varepsilon_{ih})}{r} \tag{6.7}$$

If we follow this route for the system given in equation 6.4 above then we have:

$$\begin{aligned} w_i &= \alpha_i(\mathbf{a}) + \sum_{j=1}^{n} \gamma_{ij} \ln(p_j \exp(\varepsilon_{jh})) + \beta_i(\ln(rz) - \ln(p_i \exp(\varepsilon_{ih}))) + \\ &\quad \delta_i(\ln(rz) - \ln(p_i \exp(\varepsilon_{ih})))^2 \end{aligned} \tag{6.8}$$

If we did not have a quadratic term then this would simply require adding a heterogeneity term to each equation in the usual fashion. However, the presence of the quadratic term considerably complicates matters. Thus the general approach introduces a tension between flexibility and allowing for heterogeneity. For the analysis used here it turns out to be easiest to tailor the modelling of heterogeneity to the specific form chosen.

For the functional form in equation (6.4) we included observable sources of heterogeneity in an additive way in the intercept (see (6.2) above). We can obviously follow the same route for unobserved heterogeneity:

$$\alpha_i(\mathbf{a}) = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} a_k + \varepsilon_{ih} \tag{6.9}$$

This amounts to adding an error term to each equation; we shall also assume that this error terms captures other sources of specification error such as measurement error in the left hand side variable (w_i) and functional form misspecifications. This is the approach we shall adopt in our empirical work below[6]. This highlights one considerable advantage of the functional form used here as against, say, an Almost Ideal Marshallian specification. In the latter we deflate total expenditure by a function of prices, demographics and unobservable heterogeneity. This introduces considerable complications for estimation in Marshallian systems if we wish to introduce heterogeneity in a coherent way (see McElroy (1987), Brown and Walker (1989) and Blundell, Browning and Crawford (1998)). By contrast, in our formulation we deflate the welfare measure ($=rz$) by a single price which makes things much simpler.

To complete the heterogeneity specification we need to specify the stochastic structure. One obvious assumption to make is that conditional on prices, demographics and the level of the reference good, the heterogeneity terms have zero mean:

$$E(\varepsilon_{ih} | \mathbf{a}, \mathbf{p}, z) = 0 \tag{6.10}$$

[6]We could also allow for heterogeneity in the β_i and δ_i terms with few extra complications except that this induces explicit heteroscedasticity in the error.

Although extremely convenient, this is implausible for the reference good. To illustrate, consider the very simplest case in which we have homothetic preferences, no price variation and additive heterogeneity, so that (Marshallian) expenditures are given by:

$$\begin{aligned} x_{ih} &= \alpha_i x_h + \nu_{ih} \\ x_{zh} &= \alpha_z x_h + \nu_{zh} \end{aligned} \tag{6.11}$$

where x_{ih} is the expenditure on good i by household h. Generally there will be correlations between total expenditure and the error terms due to measurement error and infrequency of purchase, so that:

$$E(\nu_{ih}|x_h) \neq 0 \tag{6.12}$$

This is the conventional reason for instrumenting total expenditure in empirical demand analysis. Almost all demand studies take household income as a valid instrument. More importantly for our purposes, the error terms will also be correlated with each other due to adding-up and heterogeneity. The adding-up constraint imposes that ($\sum_{i=1}^{n} x_{ih}+x_{zh} = x_h$) for all households so that some errors have to be negatively correlated. As regards heterogeneity, a simple example is given by 'food at home' (the reference good) and 'restaurants'. If some households like eating out and others don't then the error terms on these two goods will be negatively correlated (over and above the correlation caused by adding-up). The almost universal choice of income as an instrument implicitly assumes that this heterogeneity is uncorrelated with income.

How does this relate to heterogeneity in m-demands? To see this, invert on the reference good in (6.11) and substitute:

$$x_{ih} = \frac{\alpha_i}{\alpha_z} x_{zh} - \frac{\alpha_i}{\alpha_z} \nu_{zh} + \nu_{ih} \tag{6.13}$$

From this we see that in a simple regression of expenditures on the reference good expenditure there are two sources of correlation between the reference good and the error term. First, there is the conventional measurement error bias from using $x_{zh} = (\alpha_z x_h + \nu_{zh})$ rather than αx_h. Second, any correlation between the errors in (6.11) will cause ν_{ih} to be correlated with x_{zh}. As we have seen, such correlations could arise from adding-up or from heterogeneity. Thus to derive consistent estimates of the m-demands we have to allow for the endogeneity of the reference good. If we adopt the usual approach, this requires that we observe some measure of household incomes; this is usually the case and is so for the data used below. But can we reasonably assume that income is a good instrument? Certainly it is correlated with the level of the reference good (which is taken to be normal). As we have discussed above, the usual use of income as an instrument for Marshallian demands also validates its use here. The fundamental point is that the dispersion of households over the same budget surface is assumed independent of income (remember that households can have different incomes even if they have the same total expenditure). By instrumenting we use only variations between budget surfaces to identify the m-demand parameters. Based on this, we shall use income as an instrument for the level of the reference good in our empirical analysis below. As we shall see, this context is one in which allowing for endogeneity makes enormous differences to the inferences we draw.

6.3. The data and results.

We present here an empirical illustration using data drawn from the Canadian Family Expenditure Survey (FAMEX). The surveys we use are from 1974, 1978, 1982, 1984, 1986, 1990 and 1992. As well as the cross-time variation in prices that this induces, we also use the cross-region variation in prices that comes from different indirect tax rates and transport costs. Specifically, we consider a sample of single males who are in full time work for the whole of the survey year and are under 65 years of age. The reason for taking singles rather than, say, married couples is that Browning and Chiappori (1998) found that Slutsky symmetry was rejected for a sample of multi-person households from this data source but not for single person households. Given this, we have no reason to expect that our symmetry conditions would be satisfied for multi-person households[7] and hence we restrict attention to single person households. We model seven composite commodities: food at home; food outside the home; services; transport; recreation; vices and clothing. We take 'food at home' as our reference good. For demographics (the **a** variables in equation (6.4)) we take age, age squared and dummies for region of residence, being a homeowner, being a car owner, being a city dweller, having college education, having less than high school education, being a French speaker and having neither French nor English as a first language. These are all conventional controls for observable heterogeneity in Canadian demand studies.

We first estimate a system of six equations for the system given in equation

[7] And, in fact, the m-demand symmetry conditions do not hold for couples. The derivation of m-demands for collective models is left to future work.

(6.4) using conventional GMM techniques. We include the log of the prices deflated by the reference good price as well as the log of the reference good price itself. Thus the t-statistic on the latter provides a test for homogeneity in each equation. The instruments for the food at home quantity terms $((\ln(rz) - \ln(p_i))$ and its square) are the log of real net income and the log squared, log cubed and inverse of real net income.

To illustrate the importance of allowing for the endogeneity of food expenditures we note that the OLS relationship between restaurant expenditures and food at home is negative sloped. Taken at face value this would suggest that restaurant expenditures are an inferior good. However, the more probable cause is that here heterogeneity outweighs the income relation and we are simply observing that households that spend more on restaurants spend less on food at home. Once we instrument food expenditures we find a positive relationship (see figure 1).

We have two degrees of over-identification per equation and 12 for the system. The over-identifying (Sargan) test statistic has a value of 15.02 which has probability value of 24% under the null. We also checked for our functional form by running the system with the log cubed and the inverse of (rz/p_i) on the right hand side. In no case were these extra variables 'significant' for any of the six goods modelled; the joint test for the exclusion of these variables from the system has a $\chi^2(12)$ value of 7.8. We conclude that the quadratic form in equation (6.4) is flexible enough to capture the variation in the demands in the goods modelled. The $\chi^2(12)$ statistic for the log and log squared term in the system is 60.2 and for every good the two variables are jointly 'significant'.

Table 1 presents the parameter estimates for the unrestricted system. The only thing to note here is that in no case is the log food at home price significant. Thus homogeneity is not rejected; to reinforce this we note that the system test for homogeneity gives a $\chi^2(6)$ value of 1.9. Imposing homogeneity, we test for symmetry. The $\chi^2(15)$ value of the test for symmetry is 21.8 which has a probability of 11.2% under the null; we conclude that symmetry is not rejected. Table 2 presents the estimates with homogeneity and symmetry imposed.

Finally, we compare the fit of our m-demands with the fit implied by the Quadratic Almost Ideal demand system (QAIDS, see Banks, Blundell and Lewbel (1997)). This models budget shares as a function of demographics, log prices and the log and log squared of 'real' total expenditure. From the food at home budget share equation we can find the implied relationship between expenditures on food at home and total expenditure. Then we can substitute this into the other budget shares to give the implied relationship between the demands for 'food outside the home', services etc. and our reference good 'food at home'. This fit can then be compared to the fit from our m-demand system. There are several reasons why the two fits might diverge. First, the two represent different 'mean' Engel curves so that it is impossible for them to coincide for all values of 'food at home'. Second, the interactions between price effects and income effects for the two systems are quite different. Third, the heterogeneity structures implicit in the two systems are different. Finally, the stochastic structures implicit in using functions of real income as instruments in the two systems are different and are not mutually consistent. Despite this, however, it is important to check whether the two give similar predictions. The QAIDS is widely believed to provide a good fit to demand data so that significant divergences from it might indicate some problems with our m-demand estimates.

In Table 3 we compare the means of the m-demand fits and the QAIDS fits (both with demographics set at their mean) with the means of the original data. Remember that we have a sample of single males so these means are somewhat different from those for, say, married couples. Although neither the m-demands nor the QAIDS fit the levels of quantities directly the two set of means are reasonably close to the raw data means.

TABLE 3: Means of data and fits

	Raw data	M-demand fit	QAIDS fit
Restaurant	2.30	2.85	2.33
Services	2.04	2.41	2.38
Transport	3.75	4.01	3.40
Recreation	2.01	2.90	2.39
Vices	2.79	2.62	1.79
Clothing	1.19	1.08	1.36

All values in thousands of 1992 dollars.

For a further comparison, in figures 1 to 6 we graph the fits of quantities against the quantity of food at home from our m-demand estimates (along with a 95% confidence interval derived from these estimates) and the fit implied by the QAIDS system. Since we are concerned with comparing the shapes of the fits, we have adjusted the means of the two fits to be the same as the mean of the original data. We have also truncated the support of 'food at home' since the tail behaviour of the QAIDS is somewhat wild[8]. As can be seen from these figures, for no good is

[8] The tail behaviour of the m-demands appears superficially better but this because we restrict it to be a quadratic.

the QAIDS fit between the 95% confidence bounds for all value of food at home. Indeed, sometimes the shapes of the two fits are quite different. For example, the QAIDS 'transport' fit (see figure 3) implies that transport quantities are decreasing in food at home for 'high food' people whereas it is upwards sloping for the m-demand fit. Since the QAIDS estimates imply that transport is an inferior good for high income people, this is by no means a sign that the m-demands are 'wrong'. Nevertheless, the divergence between the two sets of fits is worrying and merits further work. In particular, it is important to identify how much of the difference comes from the fact the two systems represent different average preferences and how much comes from differences in auxiliary assumptions.

7. Conclusions.

We have presented some theory for demands that depend on prices and the quantity consumed of some other good. We showed that even if we do not observe a full set of quantities there are still conditions that empirical estimates have to satisfy. The most important outstanding issue is: how much can we (non-parametrically) identify of the utility function over all goods, $v(q_1, ..q_m, q_{m+1}, ...q_n, z)$, from m-demands for a sub-set of them $((q_1, ..q_m))$?

We have also presented an illustration of the estimation of m-demands on Canadian demand data and have shown that the theory restrictions are not rejected. On the other hand, our m-demand estimates indicate significant divergences from the m-demands implied by a widely used budget share system. More work is required to identify the source of these divergences.

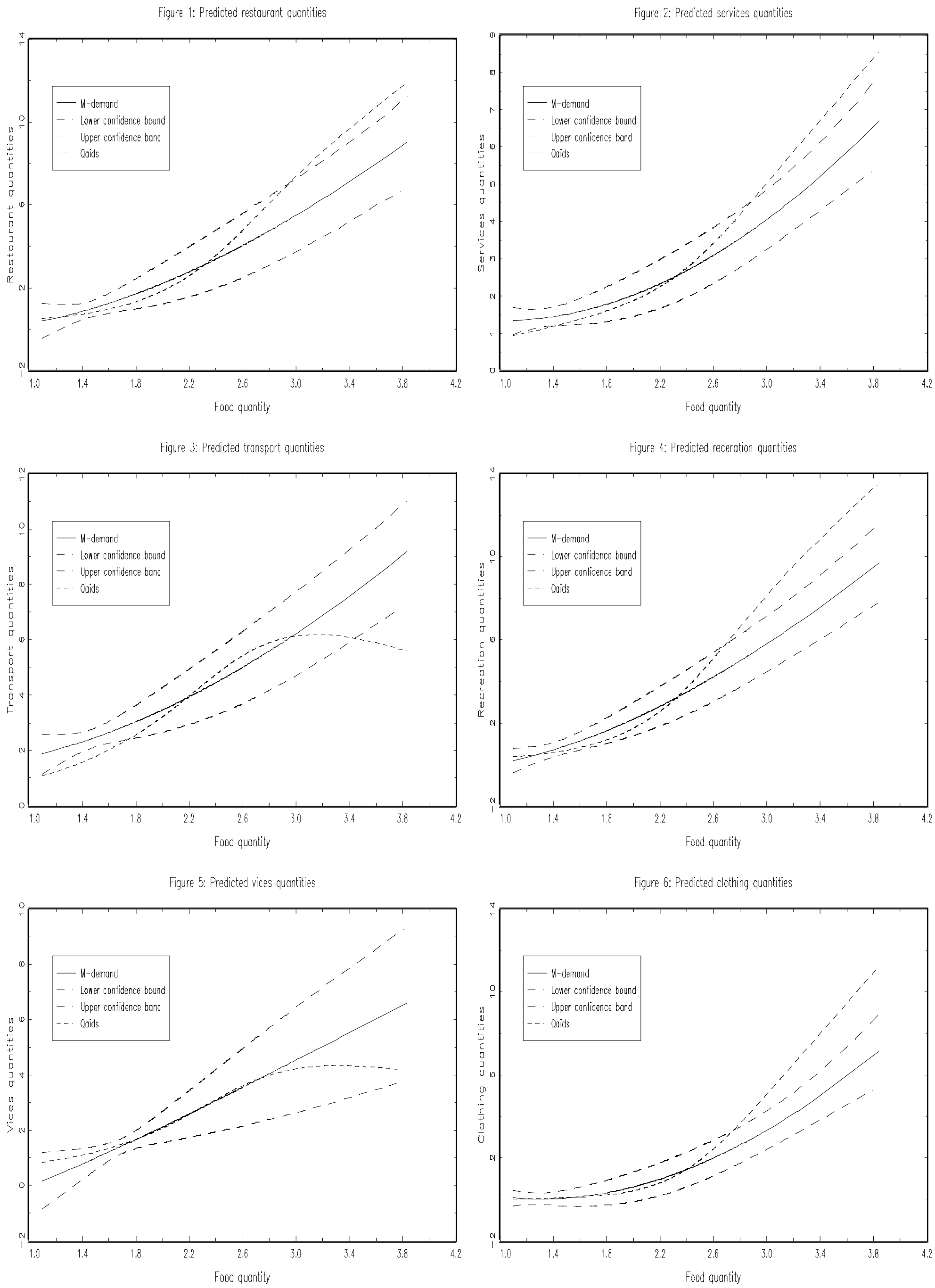
Figure 1: Predicted restaurant quantities
M-demand
Lower confidence bound
Upper confidence band
Qaids
Restaurant quantities
Food quantity
Figure 2: Predicted services quantities
Services quantities
Figure 3: Predicted transport quantities
Transport quantities
Figure 4: Predicted receration quantities
Recreation quantities
Figure 5: Predicted vices quantities
Vices quantities
Figure 6: Predicted clothing quantities
Clothing quantities

Figure 7.1:

Table 1. Single males: unrestricted estimates.

	RE	SE	TR	RE	VI	CL
INTERCEPT	-0.01 (0.59)	1.03 (0.26)	0.25 (0.50)	-0.43 (0.45)	-0.14 (0.63)	-0.13 (0.31)
HOME	-0.19 (0.11)	0.44 (0.06)	-0.17 (0.10)	-0.18 (0.09)	-0.13 (0.08)	-0.23 (0.08)
CITY	0.28 (0.11)	-0.07 (0.07)	0.09 (0.11)	0.16 (0.11)	0.13 (0.09)	0.06 (0.08)
CAR	-0.08 (0.10)	0.11 (0.05)	1.76 (0.07)	0.16 (0.09)	-0.01 (0.07)	0.04 (0.07)
REGION1	-0.04 (0.19)	0.30 (0.11)	0.29 (0.18)	-0.05 (0.18)	0.11 (0.15)	0.19 (0.13)
REGION2	-0.17 (0.20)	0.13 (0.11)	-0.05 (0.16)	-0.15 (0.18)	-0.21 (0.14)	-0.10 (0.15)
REGION4	0.31 (0.37)	0.20 (0.21)	0.01 (0.33)	0.11 (0.35)	0.21 (0.28)	0.20 (0.28)
REGION5	0.53 (0.32)	0.18 (0.17)	0.01 (0.28)	0.18 (0.28)	0.30 (0.23)	0.29 (0.22)
AGE	-0.15 (0.05)	-0.06 (0.03)	-0.22 (0.04)	-0.30 (0.04)	-0.14 (0.04)	-0.14 (0.03)
AGE2	0.03 (0.04)	0.01 (0.02)	0.07 (0.03)	0.09 (0.03)	0.03 (0.03)	0.04 (0.03)
COLLEGE	0.04 (0.10)	0.06 (0.05)	-0.10 (0.09)	0.23 (0.09)	-0.25 (0.08)	0.07 (0.07)
LOWED	-0.32 (0.18)	-0.26 (0.10)	-0.18 (0.15)	-0.25 (0.17)	-0.03 (0.13)	-0.26 (0.13)
FRENCH	0.07 (0.17)	-0.11 (0.09)	0.00 (0.14)	0.05 (0.17)	0.07 (0.12)	0.13 (0.13)
OTHERL	-0.46 (0.13)	-0.19 (0.08)	-0.23 (0.11)	-0.28 (0.13)	-0.34 (0.08)	-0.19 (0.12)
P(REST)	-0.04 (1.35)	-0.01 (0.82)	0.40 (1.33)	-1.09 (1.34)	-1.06 (1.16)	-1.42 (1.03)
P(SERV)	-0.17 (2.01)	1.68 (0.99)	-2.09 (1.81)	-0.41 (1.79)	1.49 (1.61)	-0.58 (1.36)
P(TRAN)	0.31 (1.01)	0.10 (0.56)	2.32 (0.98)	0.08 (0.92)	0.08 (0.70)	0.63 (0.71)
P(RECR)	2.47 (1.93)	-0.12 (1.03)	0.88 (1.79)	0.68 (1.71)	0.47 (1.34)	-0.36 (1.42)
P(VICE)	-0.15 (0.82)	0.38 (0.47)	-0.48 (0.85)	0.62 (0.80)	1.20 (0.77)	0.54 (0.59)
P(CLOTH)	-0.64 (1.19)	-0.46 (0.68)	-0.34 (1.05)	0.52 (1.06)	0.17 (0.79)	1.27 (0.95)
P(FATH)	0.71 (0.95)	0.12 (0.53)	0.39 (0.93)	0.82 (0.89)	0.19 (0.77)	0.03 (0.67)
QFATH	0.73 (1.75)	-1.28 (0.88)	-0.65 (1.47)	0.96 (1.31)	1.89 (1.44)	-0.85 (1.05)
QFATH2	0.62 (0.98)	1.19 (0.56)	0.84 (0.80)	0.65 (0.78)	-0.42 (0.55)	1.60 (0.73)

Standard errors given in brackets.

Table 2. Single males: homogeneity and symmetry imposed.

filler	RE	SE	TR	RE	VI	CL
INTERCEPT	0.31 (0.50)	1.11 (0.23)	0.62 (0.44)	-0.16 (0.38)	-0.26 (0.49)	0.05 (0.26)
HOME	-0.16 (0.11)	0.45 (0.06)	-0.16 (0.09)	-0.15 (0.09)	-0.09 (0.07)	-0.20 (0.08)
CITY	0.23 (0.11)	-0.10 (0.06)	0.04 (0.11)	0.14 (0.10)	0.09 (0.08)	0.06 (0.08)
CAR	-0.07 (0.10)	0.11 (0.05)	1.76 (0.07)	0.15 (0.08)	-0.00 (0.07)	0.04 (0.07)
REGION1	-0.12 (0.14)	0.27 (0.08)	0.10 (0.13)	-0.01 (0.13)	0.06 (0.11)	0.16 (0.10)
REGION2	-0.23 (0.17)	0.14 (0.09)	-0.05 (0.13)	-0.16 (0.16)	-0.27 (0.12)	-0.07 (0.13)
REGION4	0.17 (0.14)	0.17 (0.09)	-0.05 (0.13)	0.10 (0.14)	0.08 (0.09)	0.09 (0.11)
REGION5	0.23 (0.18)	0.12 (0.10)	-0.07 (0.15)	0.03 (0.15)	0.13 (0.12)	0.13 (0.11)
AGE	-0.11 (0.05)	-0.04 (0.02)	-0.19 (0.04)	-0.26 (0.04)	-0.12 (0.04)	-0.12 (0.03)
AGE2	0.00 (0.04)	-0.01 (0.02)	0.05 (0.03)	0.06 (0.03)	0.01 (0.03)	0.02 (0.03)
COLLEGE	0.07 (0.09)	0.07 (0.05)	-0.08 (0.09)	0.26 (0.09)	-0.25 (0.07)	0.10 (0.07)
LOWED	-0.32 (0.17)	-0.24 (0.10)	-0.19 (0.15)	-0.25 (0.16)	-0.05 (0.12)	-0.24 (0.13)
FRENCH	0.05 (0.17)	-0.10 (0.09)	-0.02 (0.13)	0.03 (0.16)	0.08 (0.11)	0.13 (0.13)
OTHERL	-0.43 (0.12)	-0.15 (0.08)	-0.20 (0.11)	-0.24 (0.13)	-0.32 (0.08)	-0.16 (0.11)
P(REST)	0.74 (0.68)	0.43 (0.40)	0.23 (0.30)	0.63 (0.51)	0.30 (0.22)	-0.26 (0.28)
P(SERV)	0.43 (0.40)	1.63 (0.58)	-0.26 (0.27)	0.04 (0.48)	0.37 (0.15)	-0.52 (0.25)
P(TRAN)	0.23 (0.30)	-0.26 (0.27)	1.21 (0.53)	0.01 (0.31)	0.06 (0.15)	0.09 (0.21)
P(RECR)	0.63 (0.51)	0.04 (0.48)	0.01 (0.31)	-0.47 (0.75)	0.81 (0.21)	-0.01 (0.33)
P(VICE)	0.30 (0.22)	0.37 (0.15)	0.06 (0.15)	0.81 (0.21)	0.81 (0.34)	0.24 (0.12)
P(CLOTH)	-0.26 (0.28)	-0.52 (0.25)	0.09 (0.21)	-0.01 (0.33)	0.24 (0.12)	0.78 (0.27)
QFATH	0.94 (1.45)	-0.15 (0.08)	-0.20 (0.11)	-0.24 (0.13)	-0.32 (0.08)	-0.16 (0.11)
QFATH2	0.18 (0.79)	-0.94 (0.78)	-0.86 (1.26)	0.97 (1.14)	2.71 (1.07)	-0.81 (0.94)

Standard errors given in brackets

References

[1] Altonji, Joseph G. (1986) "Intertemporal Substitution in Labor Supply: Evidence from Micro Data" *Journal of Political Economy*, 94(3), Part 2, June 1986, pages S176-S215.

[2] Attanasio, Orazio and Thomas MaCurdy (1997), "Interactions in family labour supply and its implications for the impacts of EITC", mimeo, University College London.

[3] Banks, James, Richard Blundell and Arthur Lewbel (1997), "Quadratic Engel Curves and Consumer Demand" *Review of Economics and Statistics*, 79(4), November 1997, pages 527-39.

[4] Blundell, Richard, Martin Browning and Ian Crawford (1998), "Nonparametric Engel curves and revealed preference", mimeo, Copenhagen University.

[5] Brown, D.J. and R. L. Matzkin (1997),"Estimation of nonparametric functions in simultaneous models with an application to consumer demand", mimeo, Northwestern University.

[6] Brown, B.W. and Walker, M.B., (1989), "The random utility hypothesis and inference in demand systems" *Econometrica*, 57, 815-829.

[7] Browning, Martin and Pierre-Andre Chiappori (1998), "Efficient household behaviour: a characterisation and test", *Econometrica*, November 1998.

[8] Browning, Martin, Angus Deaton and Margaret Irish (1985), "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle", *Econometrica*, 53(3), May 1985, pages 503-43.

[9] Browning, M.J. and Meghir, C. (1991), "The effects of male and female labour supply on commodity demands", *Econometrica*, 59, 925-951.

[10] Heckman, James J. (1974a), "Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle", *American Economic Review*, 64(1), March 1974, pages 188-94.

[11] Heckman, James J. (1974b), "Effect of Child-Care Programs on Women's Work Effort", *Journal of Political Economy*, 82(2), Part II, March-April 1974, pages S136-S163.

[12] MaCurdy, Thomas E. (1981), "An Empirical Model of Labor Supply in a Life-Cycle Setting" *Journal of Political Economy*, 89(6), Dec. 1981, pages 1059-85.

[13] McElroy, M. B. (1987), "Additive General Error Models for Production, Cost and Derived Demand or Share Equations" *Journal of Political Economy*, 95, 737-757.

[14] Meghir, Costas and Guglielmo Weber (1996), "Intertemporal Nonseparability or Borrowing Restrictions? A Disaggregate Analysis Using a U.S. Consumption Panel", *Econometrica*, 64(5), September 1996, pages 1151-81.

[15] Murphy, F. P. (1998), "Notes on m-demands", private communication.

[16] Pollak, Robert A. (1971), "Conditional Demand Functions and the Implications of Separable Utility", *Southern Economic Journal*, 37(4), April 1971, pages 423-33.