Discussion Papers
Department of Economics
University of Copenhagen

No. 11-01

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http://www.econ.ku.dk

ISSN: 1601-2461 (E)

Regulated competition under increasing returns to scale

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December 2010

Abstract

This paper proposes a mechanism for the regulation of firms in the context of asymmetric information with the aim to induce firms to report its private information truthfully and to save information rents. Baron and Myerson (1982) have considered this problem and derived an optimal policy for regulating a monopolist with unknown costs. They show that it was possible to create a regulatory mechanism that induced the firm to report its private information truthfully. To secure this, a part of the mechanism is to pay the firm a subsidy. This article presents a regulatory mechanism which explores competition in the context of an industry characterized by increasing returns to scale. In contrast to the model in this article, the Baron and Myerson model doesn't consider increasing returns to scale. In equilibrium each firm chooses to report truthfully without receiving any subsidy. However, the use of competition gives rise to an efficiency lost.

1. Introduction

One of the most common themes in the theory of regulation is to find method for which asymmetric information between a regulator and firms can be reduced and informational rents can be saved. Starting with Loeb and Magat (1979), regulation has been modelled as a principal-agent problem and further to be analysed in connection to the incentive compatibility where conditions are placed that induce one to reveal his information to another (see for example Myerson (1979)). Baron and Myerson (1982) considers the problem of asymmetric information and presents a mechanism that induce a firm to report its cost truthfully. A part of this mechanism is to pay the firm a subsidy which gives the firm no incentive to misreport its costs. Since Baron and Myerson, asymmetric information models have been studied in a variety of contexts. For example, work by Lewis and Sappington (1988), Laffont and Tirole (1993) and Laffont and Martimort (2002) have added greatly to the literature and our understanding of incentive regulation. In spite of the amount of literature, there have only been a few papers dealing with more than one firm. Auriol and Laffont (1992) examine a duopoly when marginal costs are private information and fixed costs are common knowledge. Their model follows the ideas of Baron and Myerson, but in a more static structure. The Auriol-Laffont mechanism makes use of yardstick competition to regulate the market. One conclusion is that asymmetric information has informational costs, but the rents are lowered by the use of yardstick competition. Others have studied mechanism design in markets of more than one firm (see Dalen (1988), Tangerås (2009), among others), but again, asymmetric information has informational rents and rents the regulator has to accept.

This paper analyse how the existence of potential competition can be used to save informational rents. By allowing competition in industries such as electricity, gas or transport, we know that the introduction of yardstick competition may reduce rents, but can we remove all informational rents? This paper present a regulatory mechanism which secure truthfully costs reports and remove informational rents, or in contrast to the Baron and Myerson mechanism without paying any subsidies. As a drawback, the use of competition give rises to an efficiency lost. In contrast to other models, among the models mentions here, our mechanism is analysed is the framework of increasing returns to scale technology, and besides yardstick competition, take use of a rate of returns on capital which is used to secure competition.

While there is a extensive literature on how to regulate an industry, mechanism design in the framework of oligopoly and increasing returns to scale technology in a would of asymmetric information is given less attention. A paper close to this paper is by Sengupta and Tauman (2004). They consider a oligopolistic market with increasing returns to scale, but derive a inventive mechanism based on a bidding contract. The paper use competition and subsidies in such a way that the regulator offers a contract to exclusively subsidize one firm. The paper shows that the winning firm produce and all other firms exit the market. In contrast to our framework, their assume full information about the market and the cost structure.

The use of increasing returns to scale technology allows for some real live situations. There are markets for firms in industries such as electricity, gas, telecommunications, water possessing increasing returns to scale technology and this is needed in the literature of regulation. In connection with this, there are questions in debates on how to make use of the potential competition in industries characterized by monopoly or natural monopolies.

Expanding the literature on regulation in relation to increasing returns to scale and real live situations, this paper, at first, provides a mechanism which induces truthfully cost reports without paying subsidies, second, gives an evaluation of the existing regulation in the market for supply obligation gas, and at last, highlight that these special advantages can be used in every areas where firms possessing increasing returns to scale technology. Three points which haven't been shown in the existing literature.

This paper is organized as follows: Section 2 explains the structure of the model and the Baron and Myerson model. In section 3 we set up the basic model for the regulated competition. This model is then extended in the sections 4 and 5 to a mechanism in order to make it comparable with the model of Section 2 and state our main results. Section 6 contains some concluding comments and indicates fields of further research.

2. The model

We consider an industry with a technology admitting increasing returns to scale. The number of firms operating in this industry may be one or several, but all have access to the same technology, defined by cost function of the form $C(q, \theta)$, where q is the level of output and θ is a parameter describing the efficiency level of the firm. In much of what follows, we shall assume that $C(q, \theta)$ is (multiplicatively) separable in its two arguments, i.e. it takes the form

$$C(q, \theta) = \theta \bar{C}(q), \tag{1}$$

where $\bar{C}$ is a fixed cost function. We shall assume that for fixed θ, the function $C(\cdot, \theta)$ is concave, so that there is nondecreasing and possibly increasing returns to scale. The parameter θ takes values in a set Θ and is assumed to be observable only by the firm.

The demand side in our model is formalized by an inverse demand function $p(q)$, determining the price $p(q)$ at which the output q can be absorbed in the market. For considerations of welfare, we shall use the function $V(q)$ defined by

$$V(q) = \int_0^\infty p(q)\, dq, \tag{2}$$

using which we may define the consumer surplus at the level of consumption q by

$$S^C(p) = V(q) - p(q)q. \tag{3}$$

We shall be interested in comparing two very different ways of regulating this market, namely one where only a single firm is operating, being a regulated monopoly, and (2) the case where there are two firms in the market, operating under regulated competition. Of the two cases, the first one has been very thoroughly investigated, at least for the case of non-increasing returns to scale, and consequently, our treatment of this case will be restricted to checking that the methods of regulating a monopoly described in Baron and Myerson (1982) can be adapted to the present situation. This will be the topic of the remaining part of this section. Then we turn to case (2) of regulated competition in the following section.

We consider a mechanism where the firm announces its type θ, and the regulator chooses a triple (r, q, s) of functions of θ, where

(i) $r(\theta$ is the probability that the firm is allowed to carry out business, taking values in the interval [0, 1],

(ii) $q(\theta)$ is the quantity which the firm is permitted to market, and

(iii) $s(\theta)$ is a subsidy paid to the firm.

We assume that the regulator has an objective of the form

$$\int_{\Theta} [(V(q(\theta)) - p(q(\theta))q(\theta))r(\theta) - s(\theta)]\; dF(\theta) + \alpha \int_{\Theta} \pi(\theta)\, dF(\theta), \tag{4}$$

where $\pi(\theta)$ is the profit of the firm,

$$\pi(\theta) = \left[p(q(\theta))q(\theta) - C(q(\theta), \theta)\right] r(\theta) + s(\theta).$$

In addition, the firm should satisfy the incentive compatibility constraint

$$\pi(\theta, \hat{\theta}) = \left[p(q(\hat{\theta}))q(\hat{\theta}) - C(q(\hat{\theta}), \theta)\right] r(\hat{\theta}) + s(\hat{\theta}) \leq \pi(\theta)$$

for all $\theta, \hat{\theta} \in \Theta$, saying that θ is the best possible message for the firm given that the true type is θ.

In the following, we assume that Θ is an interval $[\theta^0, \theta^1]$ in $\mathbb{R}$, and that $C(q, \theta)$ is C^1 with bounded first derivative $C_2'(q, \cdot)$. Then, by the envelope theorem of Milgrom and Segal (2002), we can write

$$\pi(\theta) = \pi(\theta_1) + \int_{\theta}^{\theta_1} r(t)C_2'(q(t), t)\, dt, \tag{4}$$

and the regulation policy (r, q, s) is feasible (in the sense that the firm has non-negative profits and is as well off with truth-telling as with any other reporting) if for all $\theta \in \Theta$,

$$\begin{aligned} &\pi(\theta) \geq 0, \\ &r(\theta)C_2'(q(\theta), \theta) \geq r(\hat{\theta})C_2'(q(\hat{\theta}), \hat{\theta}),\ \text{all } \hat{\theta} \geq \theta. \end{aligned}$$

To proceed, we assume that the cost function is multiplicatively separable,

$$C(q, \theta) = \theta \hat{C}(q)$$

for all q and θ, so that in particular is independent of θ.

$$C_2'(q, \theta) = \bar{C}(q)$$

We then have that the value of the objective function at the regulatory policy (r, q, s) can be written as

$$\int_{\theta_0}^{\theta_1} \left[V(q(\theta)) - \bar{C}(q(\theta))z_\alpha(\theta)\right] r(\theta) f(\theta) d\theta - (1 - \alpha)\pi(\theta_1), \tag{5}$$

where

$$z_\alpha(\theta) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}. \tag{6}$$

Indeed, using the expression for $\pi(\theta)$ and changing the order of integration, we get that

$$\begin{aligned}
\int_{\theta_0}^{\theta_1} \pi(\theta) f(\theta) &= \int_{\theta_0}^{\theta_1} \left[\pi(\theta_1) + \int_{\theta}^{\theta_1} r(t)\bar{C}(q(t))\, dt\right] f(\theta) d\theta \\
&= \pi(\theta_1) + \int_{\theta_0}^{\theta_1}\int_{\theta_0}^{t} r(t)\bar{C}(q(t)) f(\theta) d\theta dt \\
&= \pi(\theta_1) + \int_{\theta_0}^{\theta_1} r(t)\bar{C}(q(t)) F(t)\, dt.
\end{aligned}$$

Next, we notice that

$$p(q(\theta))q(\theta)r(\theta) + s(\theta) = \pi(\theta) + C(q(\theta),\theta)r(\theta), \tag{7}$$

so that

$$\begin{aligned}
&\int_{\theta_0}^{\theta_1} \left[(V(q(\theta)) - p(q(\theta))q(\theta))\, r(\theta) - s(\theta) + \alpha\pi(\theta)\right]\, dF(\theta) \\
&\quad = \int_{\theta_0}^{\theta_1} \left[(V(q(\theta)) - \bar{C}(q(\theta))\theta) r(\theta) - (1-\alpha)\pi(\theta)\right] f(\theta)\, d(\theta)
\end{aligned}$$

from which we get (5) after inserting (7).

To find the regulatory policy which maximizes (4) we make the simplifying assumption that the quantity $\dfrac{F(\theta)}{f(\theta)}$ is increasing in θ, so that also $z_\alpha(\theta)$ is increasing in θ. We then have that $q(\theta)$ must be maximizing

$$V(q) - z_\alpha(\theta)\bar{C}(q)$$

for each θ, so that $q(\theta)$ is the quantity which arises from marginal cost pricing given the cost function $z_\alpha(\theta)\bar{C}(q)$. The function $r(\theta)$ should be chosen such that $r(\theta) = 1$ when the the welfare component $V(q(\theta)) - z_\alpha(\theta)\bar{C}(q(\theta))$ is non-negative and 0 otherwise, since the objective function is linear in r. Finally, the subsidy function $s(\theta)$, which enters the objective function only through $\pi(\theta_1)$, can be defined as the smallest one which does not violate the incentive compatibility constraints, implying that $\pi(\theta_1) = 0$. Using (6) and inserting the expression for $\pi(\theta)$, we get

$$\begin{aligned}
s(\theta) &= -p(q(\theta))q(\theta) + C(q(\theta),\theta)r(\theta) + \pi(\theta) \\
&= [\bar{C}(q(\theta))\theta - p(q(\theta))q(\theta)]r(\theta) + \int_{\theta}^{\theta_1} r(t)\bar{C}(q(t))\, d(t).
\end{aligned}$$

Summing up the preceding argumentation, we have the following.

Proposition 1. *Assume that $C(q,\theta)$ is multiplicatively separable, and that the ratio $F(x)/f(x)$ is non-decreasing in θ. Then the optimal regulary policy $(r(\cdot), q(\cdot), s(\cdot))$ is given by*

(i) $q(\theta) = \mathrm{Argmax}_q \left[V(q) - z_\alpha \bar{C}(q)\right]$,

(ii) $r(\theta) = \mathrm{sgn}\left[V(q(\theta)) - z_\alpha(\theta)\bar{C}(q(\theta))\right]$,

(iii) $s(\theta) = \bar{C}(q(\theta))\theta - p(q(\theta))q(\theta)]r(\theta) + \int_\theta^{\theta_1} r(t)\bar{C}(q(t))\,d(t)$,

where $z_\alpha(\theta)$ is given by (6).

3. Regulated duopoly under perfect information

In this section, we consider a method of regulating the market which differs markedly from the case treated above, since in this case the regulator allows for more than one firm operating. In the context of increasing returns to scale, this means that an efficiency loss may have to be accepted; on the other hand, the absence of subsidies will in some sitautions make the duopoly solution preferable from a social welfare point of view.

In contrast with what was done in the previous section, we shall study a particular mechanism rather than searching for an optimal one. We shall assume that the regulatory mechanism uses a benchmarking approach based on the realized productions and associated costs. Though most regulations in practice are based on average cost, we shall make use of marginal cost, suitably defined. The argument for not using average cost is rather strong given our general background of increasing returns to scale, where marginal rather than average cost plays a role in guiding towards an welfare optimum. ome being smallest of the profits obtained in the market and the permissible profits.

We keep the setup and notation from the previous section. However, we shall proceed in several steps, beginning with the case where both firms have the same type, which then may be considered as a property of the underlying technology rather than the firm, and moreover, this type is known to the regulator. In this case, there is no problem of revelation of type, the only problem remaining for the regulator is to make sure that there the duoply is turned into a monopoly, which would be the case in an unregulated market.

Let $\theta \in \Theta$ be this common type, and define the associated cost function $\hat{C}(q) = C(q,\theta)$, which depends only on produced quantity, also assumed to be observable.

We assume throughout this section that the increasing returns considered are important but not sufficiently important to exclude that an unrestricted Cournot equilibrium with positive profits. Clearly, this equilibrium is far from being a welfare optimum, and the purpose of regulation is to approach this optimum as far as possible under the given institutional constraints.

Assumption. *There is a volume of production q such that $p(2q)q - \hat{C}(q) > 0$.*

Let q_i for $i = 1, 2$ be the choices of the two firms. A benchmark selector is a continuous map $q^* : \mathbb{R}^2_+ \to \mathbb{R}_+$ as a map which to each pair (q_1, q_2) of realized productions gives a production level in the interval $[\min\{q_1, q_2\}, \max\{q_1, q_2\}]$; we shall assume it chosen such that

$$\hat{C}'(q^*(q_1, q_2)) = \frac{\hat{C}(\max\{q_1, q_2\}) - \hat{C}(\min\{q_1, q_2\})}{\max\{q_1, q_2\} - \min\{q_1, q_2\}} \tag{8}$$

for $q_1 \neq q_2$, $q^*(q, q) = q$, so that the marginal cost at the point $q^*(q_1, q_2)$ in the case where $q_1 > q_2$ equals the slope of the chord from the point $(q_2, \hat{C}(q_2))$ to the point $(q_1, \hat{C}(q_1)$. The benchmark selector $q^*(q_1, q_2)$ then satisfies the condition

$$\begin{aligned} &\left[\hat{C}(q^*(q_1, q_2)) - \hat{C}'(q^*(q_1, q_2))\,(q^*(q_1, q_2) - q_2)\right] - C(q_2) \\ &\quad = \left[\hat{C}(q^*(q_1, q_2)) + \hat{C}'(q^*(q_1, q_2))\,(q_1 - q^*(q_1, q_2))\right] - C(q_1) \end{aligned}$$

for $q_1 \geq q_2$, stating that if firms are remunerated according to the linear approximation of the cost function at the benchmark, then the difference between the regulated income and the cost is the same for each firm.

Given the benchmark selector, and the benchmark (marginal) cost

$$c^*(q_1, q_2) = \hat{C}'(q^*(q_1, q_2)),$$

we may define the the permissible revenue for any firm at the production level q_i by

$$r(q_i; q_i, q_j) = I(q_i, q_2) + c^*(q_i, q_j)q_i, \tag{9}$$

where I is an allowance for fixed costs, defined in the case where $q_1 \geq q_2$ by

$$I(q_1, q_2) = \begin{cases} \max\{a \mid a + c^*(q, q_2)q > \hat{C}(q), q \in \mathbb{R}_+\} & \text{if } q_1 \leq \mathrm{Argmax}_q p(q + q_2)q, \\ \hat{C}(q_1) - c^*(q_1, q_2)q_1 & \text{otherwise;} \end{cases} \tag{10}$$

for the case where $q_2 > q_1$ we define $I(q_1, q_2)$ similarly with the roles of q_1 and q_2 interchanged. The profit of firm i producing q_i can then be found as

$$\pi_i^r(q_1, q_2) = \min\left\{r(q_i, ; q_1, q_2) - \hat{C}(q_i), p(q_1 + q_2)q_i - \hat{C}(q_i)\right\}. \tag{11}$$

Thus, the regulatory mechanism works through c^*, the marginal cost at the benchmark production level. This cost serves as a benchmark in a type of yardstick competition, since it is used to compute the acceptable profit, computed as the sum of an agreed return on invested capital plus coverage of variable cost using the benchmark unit cost. The acceptable profit constitutes an upper bound on the earnings of the firms, since any surplus of actual earnings over the acceptable profit will be confiscated by the regulator.

Figure 1 illustrates the mechanism defined by equations (8)-(11) where $L(q_1, q_2)$ represents the unregulated profit function. The mechanism follows the Mean Value Theorem in which there exists a point, $q^*(q_1, q_2)$, in a closed interval, for example $[q_1, q_2]$, where the slope of the tangent line to the cost function, $\hat{C}(q_i)$, is equal to the slope between the two points, q_1 and q_2. This tangent line repræsents the permissible revenue, $r(q_i; q_1, q_2)$, defined by equation (9) where allowance for fixed cost, $I(q_1, q_2)$, defined by equation (10) is the difference between the two lines. Using this mechanism provides at first two points, q_1 and q_2, where the two firms make equal profits and second a point, $q^*(q_1, q_2)$, where the firms make zero profit and where the profit increases away from this point. The dotted lines, q_1 and q_2 represent the mechanism defined by equation (11). For quantities lower than q_2, the regulation is non-binding and the firms will be regulated after $p(q_1 + q_2)q_i - \hat{C}(q_i)$. If the firms produce between q_1 and q_2, the regulation is binding and the firms will be regulated after $r(q_i; q_1, q_2) - \hat{C}(q_i)$. Finally, for quantities higher than q_1, the regulation is again non-binding and the firms will be regulated after $p(q_1, q_2)q_i - \hat{C}(q_i)$. If the firms make profit above the cap, equation (10) is the part of mechanism which secure that the firms refund the difference between the cap and the exceeding amount.

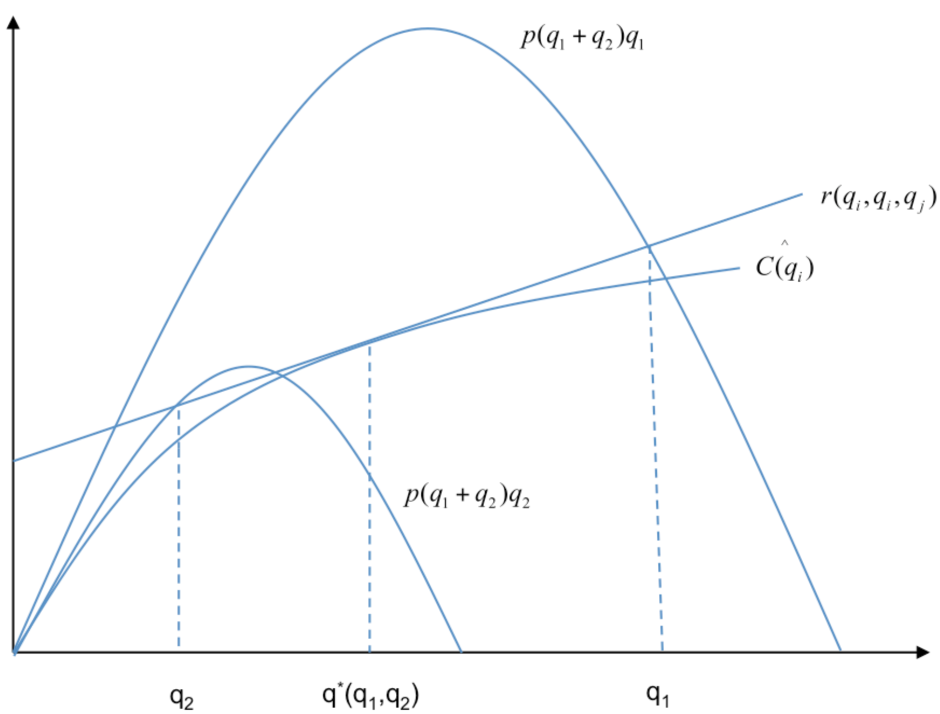

Fig. 1. Equilibrium under increasing returns to scale

We are interested in equilibria relative to this mechanism, which are pairs (q_1, q_2) such that for each $i = 1, 2$, q_i maximizes the regulated profit defined in (8).

Proposition 2. *Let (q_1, q_2) be an equilibrium with $q_1 \geq q_2$. Then*

(i) $q_1 = \max\{q_1' \mid r(q_1'; q_1', q_2) \leq p(q_1' + q_2)q_1'\}$

(ii) either $q_2 = \min\{q_2' \mid r(q_2'; q_1, q_2') \leq p(q_1 + q_2')q_2'\}$ *or q_2 maximizes* $p(q_1 + q)q - \hat{C}(q)$,

(iii) $q_1 > q_2$.

Proof: (i) Suppose that $r(q_1; q_1, q_2) > p(q_1 + q_2)q_1$. Then $\pi_1^r(q_1, q_2) = p(q_1 + q_2)q_1$ and

$$p(q_1', q_2)q_1' - \hat{C}(q_1') \leq p(q_1, q_2)q_1 - \hat{C}(q_1)$$

for all q_1' in some neighborhood of q_1. By the definition of $I(q_1, q_2)$, we then have that $r_1(q_1; q_1, q_2) = \hat{C}(q_1)$, so that $r(q_1; q_1 + q_2)q_1 \leq p(q_1 + q_2)q_1$, a contradiction, and we conclude that $r(q_1; q_1, q_2) \leq p(q_1 + q_2)q_1$.

Suppose that $r(q_1'; q_1, q_2) < p(q_1' + q_2)q_1'$ for some $q_1' > q_1$. Now for all q_1, q_2 with $q_1 \geq q_2$ and $q_1' > q_1$, we have that

$$\begin{aligned} r(q_2; q_1', q_2) - \hat{C}(q_2) &= \hat{I}(q_1', q_2) + c^*(q_1', q_2)q_2 - \hat{C}(q_2) \\ &> \hat{I}(q_1, q_2) + c^*(q_1, q_2)q_2 - \hat{C}(q_2) = r(q_2; q_1, q_2) - \hat{C}(q_2) \end{aligned}$$

since $\hat{I}(q_1', q_2) \geq \hat{I}(q_1, q_2)$ when $q_1' > q_1$ and $c^*(q_1', q_2) > c^*(q_1, q_2)$, and by our definition of $c^*(q_1, q_2)$, we have that

$$r(q_1'; q_1', q_2) - \hat{C}(q_1') = r(q_2; q_1', q_2) - \hat{C}(q_2) > r(q_2; q_1, q_2) - \hat{C}(q_2) = r(q_1; q_1, q_2) - \hat{C}(q_1),$$

contradicting that (q_1, q_2) is an equilibrium. This proves that (i) holds.

(ii) Suppose that q_2 does not maximize unrestricted profits $p(q_1 + q)q - \hat{C}(q)$. Then $r(q_2; q_1, q_2) \geq p(q_1 + q_2)q_2$, and since $r(q_2'; q_1, q_2) - \hat{C}(q_2') > r(q_2; q_1, q_2) - \hat{C}(q_2)$ for $q_2' < q_2$, we conclude that $q_2 = \min\{q_2' \mid r(q_2'; q_1, q_2) \leq p(q_1 + q_2')q_2'\}$.

(iii) If $q_1 = q_2 = \bar{q}$, then using (i) and (ii) we get that $r(q; \bar{q}, q) \leq p(\bar{q} + q)q$ only at $\bar{q}$, and $\hat{C}(\bar{q}) = p(2\bar{q})q$ which violates our general assumption, so that we may conclude that $q_1 > q_2$. □

It may be noticed that the equilibria considered above are not uniquely defined; since it is not profitable for any of the firms to increase production in an equilibrium, there may indeed by equilibria which are inferior other equilibria not only from a welfare point of view (total production is larger in another equilibrium) but also from the point of view of each of the firms. There is, however, a unique equilibrium (q_1^*, q_2^*) with the properties that $q_1^* > q_2^*$ and the total production $q_1^* + q_2^*$ is maximal over all equilibria. This particular equilibrium will be useful for us in the sequel. If we let θ vary in the interval $[\theta^0, \theta^1]$, the associated specific equilibrium defines two functions $q_1^*(\theta)$, $q_2^*(\theta)$, we we shall make use of in the sequel.

4. Duopoly with common type and asymmetric information

In the following, we relax the assumption of perfect information and assume that the regulator cannot observe the type. We retain the assumption that type is a property of the technology which is available to both firms and therefore the same for both. Also, we retain the assumption from Section 2 that $C(q,\theta)$ is multiplicatively separable,

$$C(q,\theta) = \hat{C}(q)\theta.$$

Due to the information asymmetry, each firm may however report a type which differs from the true one. As in our treatment of the monopoly situation, we shall assume that the regulatory mechanism is direct in the sense that the firm j sends a message θ_j about the type, after which the mechanism determines the quantity $q_j(\theta_1,\theta_2)$ to be produced by firm j and the net income $I_j(\theta_1,\theta_2)$ to be earned by firm j.

It seems intuitive in this situation to let determine the quantities according to the method described in the previous subsection, but using the smallest of the two messages θ_1,θ_2 as the relevant indicator of type. Define

$$\underline{\theta} = \min\{\theta_1,\theta_2\};$$

then the mechanism is given by the maps

$$\hat{q}_i(\theta_1,\theta_2) = q_i^*(\underline{\theta}),\ i = 1,2,$$

together with the rule for determining regulated income,

$$\hat{I}_i(\theta_1,\theta_2) = \begin{cases} I(q_1^*(\underline{\theta}), q_2^*(\underline{\theta})) & \theta_i \le \theta_j, \\ 0 & \theta_i > \theta_j. \end{cases}$$

Thus, the mechanism punishes the firm which sends the highest θ, redistributing income so that everything goes to the other firm, independently of whether this firm produces the larger or the smaller amount of the output commodity.

The following is a straightforward consequence of the construction.

Proposition 3. *For each type $\theta \in [\theta^0,\theta^1]$, there is an equilibrium of the mechanism $(\hat{q}_1,\hat{q}_2,\hat{I})$ with truthful revelation such that the firms 1 and 2 produce $q_1^*(\theta)$, $q_2^*(\theta)$, respectively.*

Proof: Suppose that firm j states the true value of the parameter but firm i states $\theta_i > \theta = \theta_j$. Then $\hat{q}_h(\theta_i,\theta_j) = q_h^*(\theta)$ for $h = i, j$, but $I_i(\theta_i,\theta_j) = 0 \le I_i(\theta,\theta)$, so stating a higher value is not advantageous to firm i. If firm i states $\theta_i < \theta$, then $\hat{q}_i(\theta_i,\theta) \ne q_i^*(\theta)$, and since $q_i^*(\theta)$ was defined is the production which maximizes firm i's regulated income given the cost function

$\theta\bar{C}(q)$, we have that $I_i(\theta_i, \theta) \leq I_i(\theta.\theta)$, showing that θ is the optimal message for firm i. □

The situation considered in this section, where the type is common to both firms but not observable to the regulator, may not be a very realistic one, and in the following section, we turn to the more general case where the types, and consequently the cost functions, of the two firms may differ. The mechanism considered here, which had the sole purpose of revealing the common type, will not work in that case, and it must be adapted to the new situation. This will be the done in the following section.

At this point we notice that the mechanism considered in this section still has the feature of involving no subsidies (or, more generally, involving only subsidies which are independent of the level of production) which characterized the duopoly situation of the previous section. However, in the present situation, the mechanism may produce a revenue to the regulator, coming from the confiscation of regulated income. This will happen only out of the truth-telling equilibrium, however, since in the equilibrium no punishment is called for.

5. Extending the mechanism to the case where types may differ

When cost conditions, subsumed into the type parameter θ, may differ among firms, the simple revelation mechanism of the previous section will have be revised if truthful revelation is to be maintained. For this purpose, we let the mechanism determine the role of the firms, so that the firm stating the smaller value of θ will be the one producing the large amount of output, whereas the firm with high value of the type parameter will be the small producer. Choosing as before the benchmark value of θ as the smallest of the stated types, the mechanism will then determine levels of production $q_1^*(\theta)$ and $q_2^*(\theta)$ (where the indices now refer to the selected large and small producers rather than to the original labeling of the firms) as well as income levels; we shall use the simple rule which assigns $I_i(q_1^*(\theta), q_2^*(\theta)$ to each of the firms.

Formally, the mechanism is defined by

$$i(\theta_1, \theta_2) = \begin{cases} \text{argmin}_i \theta_i & \theta_i \neq \theta_j \\ 1 & \theta_i = \theta_j \end{cases}, \qquad j(\theta_1, \theta_2) = \begin{cases} \text{argmax}_i \theta_i & \theta_i \neq \theta_j \\ 2 & \theta_i = \theta_j \end{cases} \tag{12}$$

$$\bar{q}_{i(\theta_1,\theta_2)}(\theta_1, \theta_2) = q_1^*(\min\{\theta_1, \theta_2\}), \qquad \bar{q}_{j(\theta_1,\theta_2)}(\theta_1, \theta_2) = q_2^*(\min\{\theta_1, \theta_2\}), \tag{13}$$

$$I_{i(\theta_1,\theta_2)} = I_1(q_1^*(\min\{\theta_1, \theta_2\})), \qquad I_{j(\theta_1,\theta_2)} = I_2(q_1^*(\min\{\theta_1, \theta_2\})). \tag{14}$$

Here the functions $i(\theta_1, \theta_2)$ and $j(\theta_1, \theta_2)$ selects the large and the small producer given their stated types, with the convention that firm 1 becomes the large producer when the stated types are identical. Having selected producers, the choice of production is performed as previously, taking the smallest of the stated types as the benchmark, and regulated incomes are determined as in Section 3.

While incomes are identical, profits are not, since the small producer with type θ_2 has cost $\theta_2\bar{C}(q_2^*(\theta)) > \theta\bar{C}(q_2^*(\theta))$. If the difference between θ_2 and θ_1 is big enough, the small producer may experience negative profits, so that producing is no longer individually rational. To avoid this, we shall restrict attention to cases where the types, though different, are sufficiently close so that no producer will encounter negative profits. Formally, we state the assumption as follows.

Assumption 3. *The type parameters θ_1 and θ_2 are selected from a subset D of $[\theta^0, \theta^1]^2$ such that*

$$(\theta_i, \theta_j) \in D \Rightarrow \min_{h=i,j} I_h(q_1^*(\min\{\theta_i, \theta_j\}), q_2^*(\min\{\theta_i, \theta_j\})) - \max\{\theta_i, \theta_j\}\bar{C}(q_2^*(\min\{\theta_i, \theta_j\})) \geq 0.$$

The assumption says that the net profit of the producer which has the largest value of the type parameter, and which consequently is set to produce the small amount of output, remains nonnegative for all admissible parameter values (θ_1, θ_2). In the interpretation, this means that the industry-determined uncertainty about productivity is more important than the individual differences in productivity.

With respect to the information asymmetry we shall retain the assumption that types are not observable by the regulator, but observable to the firms. This may not be entirely satisfactory, but it allows us to keep the model simple, allowing for the particular strategy choices shown below to constitute an equilibrium.

We may now state a version of Proposition 3 which holds for the present version of the model with moderate type differences among firms.

Proposition 4. *Let the set of admissible pairs of type parameter values belong to a set D satisfying Assumption 3. Then for each (θ_1, θ_2), the strategy pair $(\max\{\theta_1, \theta_2\}, \max\{\theta_1, \theta_2\})$ is an equilibrium of the mechanism $(i, j, \bar{q}, \bar{I})$ given by (12)-(14).*

Proof: Suppose that firm j follows the equilibrium strategymax$\{\theta_1, \theta_2\}$ and let firm i choose $\theta_i' \neq \max\{\theta_1, \theta_2\}$. There are two cases to consider:

(1) $\theta_i \geq \theta_j$. Then j has chosen strategy θ_i.

If $\theta_i' < \theta_i$, then $i = i(\theta_i', \theta_i)$, so that i becomes the large producer. Consequently, firm i gets net income

$$I_1(q_1^*(\theta_i')) - \theta_i\hat{C}(q_1^*(\theta_1')) < I_1(q_1^*(\theta_i)) - \theta_i'\hat{C}(q_1^*(\theta_i)) < I_1(q_1^*(\theta_i)) - \theta_i\hat{C}(q_1^*(\theta_i)$$

since the slope of the affine function determining $I_1(q_1^*(\theta))$ is an increasing function of θ, $q_1^*(\theta_j) < q_1^*(\theta_1')$, and $\theta_i' < \theta$. This shows that the deviation is not advantageous in the case

$i = 1$. If $i = 2$, we notice that

$$I_2(q_1^*(\theta_i)) - \theta_i \hat{C}(q_2^*(\theta_i)) = I_1(q_1^*(\theta_i)) - \theta_i \hat{C}(q_1^*(\theta_i))$$

so that also in this case the deviation does not result in a higher net profit.

If $\theta_i' > \theta_i$, then production and regulated income is unchanged, so net income cannot be improved in this case.

(2) $\theta_i < \theta_j$, so that $\max\{\theta_1, \theta_2\} = \theta_j$. If $\theta_i' < \theta_j$, then as before we have that

$$I_1(q_1^*(\theta_i')) - \theta_i \hat{C}(q_1^*(\theta_1')) < I_1(q_1^*(\theta_i)) - \theta_i \hat{C}(q_1^*(\theta_i)),$$

and if $\theta_i' \geq \theta_j$, then the mechanism chooses the same production and income as with θ_i. We conclude that $(\max\{\theta_1, \theta_2\}, \max\{\theta_1, \theta_2\})$ is indeed an equilibrium. □

Although we do not in the present case obtain truthful revelation of preferences, the equilibrum exhibited achieves something which comes rather close, given the assumption that types differ only slightly among firms. The intuition behind the result is rather straightforward. The mechanism creates competition and gives the regulator the opportunity to use benchmarking. The benchmark gives the opportunity to compare performances and efficiency across firms. So in a market with asymmetric cost and increasing returns to scale technology, firms subject to benchmarking cannot credibly understate productivity and perform against the expected. And in this market, the regulator will expect the inefficient firm to produce less than the efficient firm. Following the word of Tangerås (2003), one may say that “yardstick competition filters out industry-specific productivity and thus reduces firms’ informational advantage”.

The mechanism presented in this paper makes use of this and using the efficient firm as benchmark secure truthfully cost reports without paying subsidies. This is secured because the mechanism doesn’t let the inefficient firm influence the variables used in the benchmark. This and the use of punisment/loss if a firm misreport its cost secure that the efficient firm, like the inefficient firm, has no other interest than to report truthfully. This is a essential feature and the main advantage of our mechanism: It requires minimal information for truthfully information. The regulator can apply for information and no firm is interested in misreporting. Another advantage of this mechanism is that it is no vulnerable to collusion, because the efficient firm wins nothing by a collusion. The firm’s profit is already maximised. A collusion can help the inefficient firm if the benchmarking unit cost increases, but for the efficient firm will higher unit cost, if revenue do not follow, mean lower profit. Therefore, the efficient firm will not participate in a collusion. In spite of this, our model has a drawback as the use of competition gives rise to an efficiency lost. The efficiency lost is the result of having more than one firm in the market. Thus, the regulator can therefore choose the efficient solution and have the most efficient firm produce paying a subsidy to secure truthfully cost reports or the regulator can choose competition and save subsidies, but have to accept a efficiency lost. Both solutions are costly from a welfare point of view.

6. Concluding comments: Comparing regulated monopoly and duopoly

In the preceding sections, we have discussed very different methods of regulating production in a market with nondecreasing returns to scale, namely (a) the by now classical method of truthful revelation by a monopolist, and (b) an alternative approach using regulated competition between two firms. The presence of nondecreasing returns to scale was important for the duopoly mechanism; however, it entails a potential welfare loss when there are important increasing returns to scale, since splitting total production by a single firm into production in two firms means that the cost of obtaining the same output must increase.

However, in cases where the increasing returns are not very important, this efficiency loss may turn out to be smaller than the cost of providing the right incentives for a monopolist. This can be shown by a very simple example with constant returns to scale. Suppose that the demand in the market is given by

$$p(q) = 10 - q,$$

and that we have the simple cost function

$$C(q, \theta) = \theta 2q.$$

The parameter θ is assumed to be uniformly distributed in the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$. Looking specifically at the value $\theta = 1$ we have that classical welfare optimizing production (where price equals marginal cost) is acheived at $q = 8$.

In the case of regulated monopoly, consider the case $\alpha = 0$; then we have that

$$\frac{F(\theta)}{f(\theta)} = \frac{1}{2},\ z_\alpha(1) = \frac{3}{2},$$

and the production chosen by the mechanism at $\theta = 1$ is $q = 7$. We have thus a smaller production reflecting the societal cost of obtaining truthful revelation.

Considering now the duopoly solution (with common parameter determined solely by industry conditions), we notice that the productions $q_1 = 7$, $q_2 = 0.5$ satisfy the equilibrium conditions at $\theta = 1$. Indeed, at $q_1 = 7.5$ the cost of production is 15 which is equal to the revenue to firm 1 given that firm 2 produces $q_2 = 0.5$, namely $(9.5 - 7.5) * 7.5 = 15$. And for the small producer, we similarly have that cost at q_2 is 1 which equals revenue $(2.5 - 0.5) * 0.5 = 1$. Due to our special situation of constant returns to scale, the production plan of firm 2 does not satisfy Proposition 2(ii), but it comes closer to the welfare optimum than does the regulated monopoly.

Since the comparison was done for only a single value of the parameter, the result should not be taken seen as an indication that a duopolistic regulated monopoly is preferable to the regulated monopoly. Indeed, the welfare loss connected with producing in too small a scale may very well be more important than the cost of providing incentives to the monopolist.

It should also be noticed that we have in general assumed that the duopolists may charge the same prices in the equilibrium, which may not be quite satisfactory from the point of view of applications, although some such cases do occur. An alternative could be the use of two-part tariffs, which indeed corresponds to the way that regulated income is defined, and then interpret the revenues as pertaining to the unit prices alone. A more detailed treatment of the duopoly case with the specific treatment of such features will be the topic of future research.

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