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Longevity and Schooling: The Case of Retirement

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Abstract

It is often conjectured that higher life expectancy leads to longer schooling. The reasoning behind this notion is that a longer lifespan increases the recovery period of human capital investment and thus, makes it more profitable to invest in education. This notion goes back to Ben-Porath (1967) and is therefore often termed the Ben-Porath mechanism. However, the original Ben-Porath mechanism concerns the length of economic life and not the length of life per se. This distinction is important in the presence of retirement and especially so as earlier retirement ages are observed in many western countries. This paper presents an overlapping generations model including both an educational and a retirement decision, thereby being able to test the Ben-Porath mechanism using the correct definition of length of working life. It is found that an increase in life expectancy does not necessarily increase the expected length of economic life as also early retirement can occur. Schooling still increases, however not due to the increase in the recovery horizon but due to an increase in the probability of surviving the recovery period.

JEL classification: D91, I20, J10, J26
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1 Introduction

It is often conjectured that higher life expectancy leads to longer schooling. The idea is that a longer time horizon increases the recovery period of human capital investment, thereby making it more viable to invest in education. Going back to the seminal work by Ben-Porath (1967), this idea has conventionally been termed the Ben-Porath mechanism.[1] The mechanism has been widely used in growth theory and related literature, see for example Kalemli-Ozcan, Ryder and Weil (2000) or Heijdra and Romp (2009a). Indeed there is a strong correlation between life expectancy and schooling. Over the last decades life expectancy in the United States has risen from an average of below 50 years in the beginning of the 20th century to almost 80 years today. Over the same time span, also average educational attainment in western countries has increased significantly. In the United States average years of schooling have risen from below 8 years to above 12 years in the 50 years from 1950 to 2000 alone (Barro and Lee, 2000).

Nevertheless, it has been difficult to confirm a causal relation between mortality and human capital investment empirically. Acemoglu and Johnson (2007), for example, cannot identify such causal relation. Cervellati and Sunde (2009), on the other hand, provide evidence in this direction, as do Jayachandran and Lleras-Muney (2009). In general, the empirical evidence is inconclusive and effects do not seem to be very strong.

The above mentioned theoretical work takes the number of years one can expect to live to have an effect on human capital investment, implicitly assuming that one works until death. The original Ben-Porath mechanism, however, concerns the length of work life and not the length of life per se. As the idea is that a longer work life increases the recovery horizon of human capital investment, it is the expected number of years spent in the workforce that is the relevant measure for the recovery horizon. Only looking at life expectancy would presuppose that an increase in longevity in fact increases work life. This will not necessarily be the case, as it is common to retire at the end of the life-cycle. Taking retirement into account, the expected number of years spent working might increase, stay the same or even decrease as life expectancy increases. Theoretically it has been shown that the effect of life expectancy on the retirement age is ambiguous, see for example Bloom, Canning, Mansfield and Moore (2007) or Kalemli-Ozcan and Weil (2010). In fact, in the United States the labor force participation rate of men between 60 and 64 years of age has fallen from 83 percent in 1957 to approximately 58 percent today (Hurd, 2008). Implicitly assuming an increase in working life as life expectancy increases seems, thus, not viable.

This paper therefore examines the Ben-Porath mechanism under the presence of an endogenous retirement decision. Boucekkine, de la Croix and Licandro (2002) present a model in which agents are faced with both a choice of education and of retirement. In addition, a survival function which is concavely decreasing in age is specified, making the demographics in the model more realistic. The results of this set-up confirm the Ben-Porath mechanism in that education is increased upon an increase in life expectancy. In this model this automatically increases the retirement age, as the retirement age is a multiple of the schooling

[1] This terminology will also be used in the following.

length. Thus, earlier retirement cannot occur upon an increase in life expectancy. The inclusion of an educational as well as a retirement phase is possible by using a linear utility function, which implies that individual consumption paths are indeterminate. The present paper extends this important contribution by assuming a concave utility function and shows that the results concerning the retirement decision are largely affected.[2]

Also Hazan (2009) presents a model including both education and retirement. However, lifetime here is certain. The theoretical results show that an increase in lifetime will lead to an increase in schooling but implied lifetime labor supply is unchanged, such that the retirement age increases. It is then shown empirically that this finding cannot be reconciled with actual lifetime labor supply as seen for men born in the U.S. over a period of 1840 to 1930. In fact, lifetime labor supply for men of these cohorts has actually fallen over the mentioned time period. It is therefore concluded that the Ben-Porath mechanism cannot be responsible for the observed increase in educational attainment since it implicitly implies a proportional increase in the retirement age. Section 4 of the present paper, however, shows that, taking uncertainty into account, educational attainment increases upon an increase in life expectancy even if the retirement age is reduced. Thus, a falling lifetime labor supply and increasing educational attainment can be reconciled.

Related to the problem of this paper is the work by Sheshinski (2009) and Cervellati and Sunde (2010) as well as Hansen and Lønstrup (2011) and Strulik and Werner (2012). Using a general survival function, Sheshinski (2009) shows that mortality improvements which are concentrated at older ages lead to an increase in schooling and to later retirement. Mortality improvements which are concentrated at younger ages, however, may lead to both an increase or a decrease in schooling, for a given retirement age. By considering mortality improvements taking place at all ages coupled with positive discount rates, the model presented in the paper at hand, however, is able to reproduce the empirically relevant case of increasing educational attainment and earlier retirement, which is not possible with the model presented in Sheshinski (2009). Cervellati and Sunde (2010) show that the conclusions in Hazan (2009) do not hold generally, as the necessary condition for schooling to increase is that the benefits of schooling increase relative to the costs and not an increase in total hours worked. Although the conclusions are similar, the focus of the paper at hand is quite different than that of Cervellati and Sunde (2010), the focus of the model presented here being on the explicit modelling of the retirement decision and the interaction with the schooling decision in the case of increasing longevity. Hansen and Lønstrup (2011) show that when credit markets for the young are absent it can be optimal to increase schooling and reduce the retirement age at the same time, thereby presenting a different kind of mechanism than the paper at hand. Strulik and Werner (2012) show in a three-period overlapping generations model that the length of active life can decrease upon an increase in life expectancy if the labour supply elasticity is high enough.

Assuming a constant instantaneous mortality rate, this paper presents an attempt to ex-

[2]A similar attempt was made in Echevarria (2004), however, the interaction between the life-cycle decisions of education and retirement was not taken into account.

tend and possibly fill some gaps of the above outlined research, by presenting a joint model of education and retirement under uncertain lifetimes. It will be shown that schooling unambiguously increases upon an increase in life expectancy, however not due to the Ben-Porath mechanism, as expected work life not necessarily increases. The increase in schooling is rather due to reduced uncertainty of surviving the recovery period of the educational investment. Early retirement can occur, which confirms the importance of taking retirement into account when using the Ben-Porath mechanism in endogenous growth models. One cannot be sure that an increase in life expectancy also increases lifetime labor supply. Nevertheless, the reduction in survival uncertainty seems to be enough to induce higher human capital investment. Thus, it is the increase in the expected length of work life that causes a positive relation between life expectancy and schooling.

Clearly, this increase in schooling does not imply that the growth effect of increasing life expectancy must be positive, as was often concluded by earlier work. Whether the increase in productivity of the labor force (due to increased education) is large enough to offset a possible decrease in the size of the labor force (caused by longer schooling and early retirement) is remains to be shown. This might also help to explain empirical difficulties to reconcile a positive effect of life expectancy on economic growth across countries.

From here, the paper proceeds as follows. The theoretical model is presented in section 2 and sections 3 and 4 review the main results. Section 3 discusses the analytical effects of increased life expectancy on life-cycle decisions, whereas section 4 presents numerical simulations of the main results. Section 5 concludes.

2 The Model

The model presented here is based on Blanchard's perpetual youth model (Blanchard, 1985). In this continuous-time OLG model agents face a positive probability of dying, which is constant through life and the same for all agents. In the original model, agents start working at birth and continue working until death. Here, a more realistic description of the life-cycle is incorporated. Agents can choose to postpone labor market entry through education, then being more productive, and can choose to leave the labor force earlier by retiring at the end of the life-cycle.

2.1 Demography

The description of demography is the same as in the original Blanchard model. Thus, all agents face the same, constant probability of dying and the population is assumed to be stationary. This means that the birth as well as the mortality rate, and thereby also population growth rate, have been constant for a long time. The instantaneous probability of dying will be denoted by μ in the following. Although a death rate which is constant across ages may not be very realistic, it is useful to analyze a change in mortality which takes place with the same magnitude for all ages. As the increase in life expectancy, which has taken place in western countries over the last century and which is of interest here, can be classified as such a change in mortality, the simple death rate can be justified (see also Kalemli-Ozcan et al., 2000). Additionally, agents are in fact born as adults, i.e. when they take decisions concerning education and working life. As it is adult mortality that is the relevant parameter here, the assumption of a constant mortality rate for all adults might not be so inaccurate. The model concerns long-run developments, therefore not including a maximum attainable age in the demographic description seems reasonable. This is especially true since the evidence provided by Oeppen and Vaupel (2002), who report that estimated maximum life expectancies have continuously been crossed over the past 160 years.

Denoting the probability of surviving from the time of birth, s, to time t by $P(t-s)$, this probability can be written as

$$P(t-s) = e^{-\mu(t-s)} \tag{1}$$

Population grows exponentially at rate n. Thus, the size of the total population at time t is given by

$$N(t) = N(0)e^{nt} \tag{2}$$

where N(0) is the size of the initial population at time 0. It is assumed that the flow of newborns is proportional to the existing population at every instant of time, i.e.

$$N(s,s) = bN(s) \tag{3}$$

Here, b is the birth rate. As the population is assumed to be stationary, the population growth rate will be the difference between the birth and the mortality rate. Thus, $n = b - \mu$ will hold at every point in time. The size of a cohort, born at time s and still alive at time t, is then given by

$$N(s,t) = bN(0)e^{ns}e^{-\mu(t-s)} \tag{4}$$

Agents first receive education and only thereafter enter the labor market from which they later retire. Note that, in this set-up of the model, agents will always find it optimal to choose a period of full-time education at the beginning of lige, followed by a period of full-time work and finally retreating completely from the labour market. As for education, this is due to the fact that the opportunity cost of studying is lowest in the beginning of life. The foregone wage will be lower in earlier periods as wages grow at the rate of technological progress. Also the retirement decision will optimally be taken once-and-for-all since it is determined by a comparison of the marginal utility of consumption against the marginal disutility of working and the disutility is linearly increasing in age.

2.2 Firms

Markets are assumed to be perfectly competitive. The production function of firms is assumed to have neoclassical properties according to

$$Y(t) = F(K(t), A(t)H(t)) \tag{5}$$

where K(t) is the aggregate capital stock, A(t) is a productivity factor, which is increasing with time, and H(t) is aggregate human capital at time t. More specifically, technology is growing exponentially at rate γ, i.e.

$$A(t) = A(0)e^{\gamma t} \tag{6}$$

and aggregate human capital is given by

$$H(t) = h(t)L(t) \tag{7}$$

where h(t) is average human capital and L(t) denotes the size of the workforce. Aggregating over the workforce only, implies that human capital first becomes accessible after finishing school and is 'depreciated' instantly upon retirement. Here, human capital evolves according to

$$h(t) = \begin{cases} \beta T(s)^{\alpha} & , for \quad s + T(s)^* \leq t \leq s + R(s)^* \\ 0 & , for \quad t \notin [s + T(s)^*; s + R(s)^*] \end{cases} \tag{8}$$

where $\alpha, \beta > 0$ and T(s)* and R(s)* represent the equilibrium ages of finishing education and leaving the labor force as chosen by the individual, which will be discussed later in more detail.

The economy is assumed to be a small open economy. An implication of this assumption is that the interest rate will be exogenously given by the world market. Additionally, this rate is assumed to be positive and constant, denoted by r in the following. As all firms are faced with this same interest rate and are symmetric, all firms will choose the same capital intensity, $\tilde{k}(t) \equiv \frac{K(t)}{A(t)H(t)}$. With a constant interest rate, the chosen capital intensity will also be constant and can be denoted by $\tilde{k}^*$ for all firms. In intensive form the production function can then be written as

$$\tilde{y}(t) = \frac{Y(t)}{A(t)H(t)} = F\left(\frac{K(t)}{A(t)H(t)}, 1\right) = f(\tilde{k}(t)) = f(\tilde{k}^*) \tag{9}$$

Firms are profit maximizing and profits are given by

$$\Pi(t) = Y(t) - w(t)L(t) - (r + \delta)K(t) \tag{10}$$

where $r > 0$ is the interest rate, $\delta > 0$ is the depreciation rate of physical capital and $w(t)$ is the wage rate at time t. Both labor and physical capital are paid the value of their marginal products. With no adjustment costs of capital and labor and no adjustment time, the maximization of profit at every instant in time is the same as maximizing the present value of all future profits. The marginal product of labor is then given by the RHS of (11).

$$\frac{\partial \Pi(t)}{\partial L(t)} = 0 \Rightarrow w(t) = \tilde{w}^* A(t) h(t) \tag{11}$$

defining $\tilde{w}^* \equiv f(\tilde{k}^*) - \tilde{k}^* f'(\tilde{k}^*)$. Thus, $w(t)$ is the wage for a worker with average human capital, h(t). The marginal product of capital is given by the RHS, plus the depreciation rate of capital, of (12).

$$\frac{\partial \Pi(t)}{\partial K(t)} = 0 \Rightarrow r = f'(\tilde{k}^*) - \delta \tag{12}$$

defining the capital intensity, $\tilde{k}^*$, as the interest rate is given exogenously. Capital is rented from insurance companies, which is described in more detail in the next section.

2.3 Insurance Companies

Insurance companies play an important role in two respects in this economy. Firstly, they issue negative life insurance contracts in which agents will place all their savings.[3] These contracts are bought by the agents for one unit of account at some point in time. Thereafter, they pay an interest of the actuarially fair rate every period, whereas at death the financial wealth of the agent is transferred to the insurance company. The actuarially fair rate is comprised of the risk-free rate, given by the world interest rate, plus an actuarial bonus. The insurance company invests all its financial assets in the firms, who are producing the consumption good. This investment is assumed to give a return of the risk-free rate. This set-up of negative life-insurance is as in the original Blanchard model.

Here, however, insurance companies additionally issue loans to students in order to finance their education. Due to the specification of preferences agents do not leave bequests and education therefore needs to be financed by borrowing. It is assumed that insurance companies demand the actuarially fair rate as interest on these loans.

The workings of the insurance and loans market is illustrated in Figure 1 below. Here, Z(t) is net liabilities of the insurance companies, given by deposits D(t) less student loans B(t), i.e. $Z(t) \equiv D(t) - B(t)$. The actuarial bonus is, for now, denoted by $\bar{r}$.

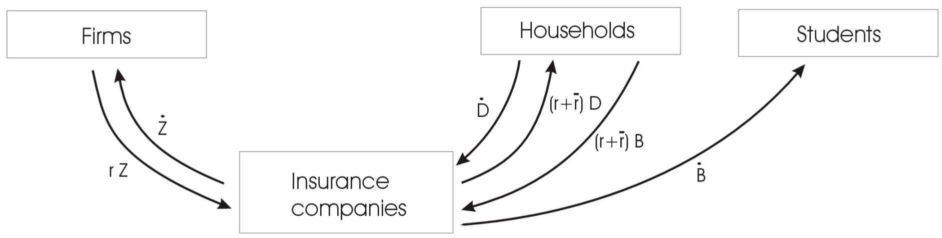


Figure 1: The insurance and loans market

[3]This was first shown by Yaari (1965) and is due to the fact that agents do not leave bequests according to the specification of their preferences.

In order to calculate the actuarial bonus, first note that profits of insurance companies at time t, are given by

$$\Pi(t) = rZ(t) - (r+\bar{r})D(t) + (r+\bar{r})B(t) + \mu(L(t)+E(t))\frac{D(t)}{L(t)+E(t)} - \mu S(t)\frac{B(t)}{S(t)} \quad (13)$$

where L(t) is the working population, E(t) is the number of retirees and S(t) denotes the student population. Note that only working agents and retirees possess financial wealth. Therefore, only upon their death, is financial wealth transferred to the insurance companies. Assuming that the insurance company is large, thereby holding deposits from a large number of agents and giving loans to a large number of students, this transfer will equal average financial wealth, equal to $\frac{D(t)}{L(t)+E(t)}$. The death of a student, on the other hand, represents a loss to the insurance company as the student's loan will not be paid back. Again assuming the insurance company is large, it looses the amount of an average student loan, given by $\frac{B(t)}{S(t)}$, upon the death of a student. As the insurance and loans market is assumed to operate under perfect competition, profits will be zero. This allows to calculate the actuarial bonus.

$$\Pi = 0 \quad \Leftrightarrow \quad \bar{r} = \mu \quad (14)$$

In equilibrium, the actuarial bonus is thus equal to the mortality rate, as in the original Blanchard model and as was first shown by Yaari (1965). That this result also holds in the extended modeling of insurance companies presented here, is due to the fact that students have the same probability of dying as all other agents, as well as the assumption that insurance companies demand the actuarially rate on student loans. Then, for a large insurance company, the actuarial bonus will exactly make up the loss of students who die before paying back the loan. The fact that surviving agents pay accumulated interest equal to the actuarially fair rate on their student loans once they start working, leads to an expected return on these loans of exactly the risk-free rate.

2.4 Households

An agent chooses a consumption path, how much time to spend in education as well as when to retire. The retirement decision is modeled by agents incurring disutility from working as well as from studying. It is assumed that this disutility is increasing in age as it becomes more tiresome for the agent to study or work, the older he gets. Another interpretation would be that the agent values free time during the entire life, but even more so at older ages. Furthermore, it is assumed that the utility function is additively separable in consumption and labor supply. Moreover, utility is assumed to be logarithmic. This eases the analysis, as the retirement decision does not depend on the level of the wage rate under this assumption. Then, the expected lifetime utility of an agent born at time s, and as seen from the time of birth, is given by

$$U(s) = \int_s^{\infty} ln(c(s,t))e^{-(\rho+\mu)(t-s)}dt - \int_s^{s+R(s)} \epsilon \cdot (t-s)e^{-(\rho+\mu)(t-s)}dt \tag{15}$$

where $\rho > 0$ is the pure rate of time preference. The term $\epsilon \cdot (t-s)$ reflects the disutility derived from work or study, where $\epsilon > 0$ is a constant disutility parameter and disutility is thereby linearly increasing in age.

Agents can either consume or save their labor income for later consumption. Thus, the agent's intertemporal budget constraint (IBC), as seen from time of birth s, is given by

$$\int_s^{\infty} c(s,t)e^{-(r+\mu)(t-s)}dt \leq a(s,s) + \int_{s+T(s)}^{s+R(s)} w(t)e^{-(r+\mu)(t-s)}dt \tag{16}$$

where $a(s,s)$ is the agent's initial financial wealth and $w(t)$ is labor income at time t.

The agent chooses a value for T(s) and R(s), i.e. decides length of education and retirement age, as well as a time path of consumption. These decisions are taken such as to maximize utility, (15), subject to the budget constraint, (16). The Lagrangian for this maximization problem is then given by

$$\begin{aligned} \mathscr{L}(s) = &\int_s^{\infty} ln(c(s,t))e^{-(\rho+\mu)(t-s)}dt - \int_s^{s+R(s)} \epsilon \cdot (t-s)e^{-(\rho+\mu)(t-s)}dt \\ &- \lambda\left(\int_s^{\infty} c(s,t)e^{-(r+\mu)(t-s)}dt - a(s,s) - \int_{s+T(s)}^{s+R(s)} \beta T(s)^{\alpha} A(t)\tilde{w}^* e^{-(r+\mu)(t-s)}dt\right) \end{aligned} \tag{17}$$

where the first-order conditions with respect to the three decision variables are

$$\frac{\partial \mathscr{L}(s)}{\partial c(s,t)} = 0 \quad \Rightarrow \frac{1}{c(s,t)}e^{(r-\rho)(t-s)} = \lambda \tag{18}$$

$$\frac{\partial \mathscr{L}(s)}{\partial T(s)} = 0 \quad \Rightarrow T(s) = \frac{\alpha}{r+\mu-\gamma}\left(1 - e^{-(r+\mu-\gamma)(R(s)-T(s))}\right) \tag{19}$$

$$\frac{\partial \mathscr{L}(s)}{\partial R(s)} = 0 \quad \Rightarrow \epsilon \cdot R(s) = \lambda w(s+R(s))e^{-(r-\rho)R(s)} \tag{20}$$

Here, it is assumed that $r + \mu > \gamma$ to ensure a meaningful equilibrium. The transversality condition, $lim_{t\to\infty} a(s,t)e^{-(r+\mu)t} = 0$, ensures an equilibrium trajectory. (19) can be interpreted as the condition that the length of education is chosen such that the marginal cost of education, time and thus the foregone wage, is equal to its marginal benefit, the discounted stream of extra earnings, $\frac{\alpha}{r+\mu-\gamma}$, adjusted for the length of the working life, R(s)-T(s). (20) gives the intuitive condition that, at the date of retirement, the disutility of work should equal the marginal utility of extra consumption financed by the additional wage income if the agent kept on working.

By differentiating (18) with respect to time, we see that the growth rate of individual consumption is given by the Keynes-Ramsey rule. Thus,

$$\frac{\dot{c}(s,t)}{c(s,t)} = r - \rho \equiv g \tag{21}$$

for any T(s) and R(s). Assuming that $r > \rho$, which implies that the economy is populated by relatively patient agents, this growth rate will be positive. Note that this growth rate does not depend on the mortality rate. This is due to the fact that agents can fully insure themselves against the risk of dying by purchasing negative life insurance. Furthermore, it is worth noting that the consumption growth rate is the same for all agents, independent of the time of birth.

Note that the initial level of consumption can now be written as

$$c(s,s) = (\mu + \rho) \cdot v(s,s) \tag{22}$$

where human wealth at time of birth, v(s,s), can be simplified to

$$\begin{aligned} v(s,s) &= \int_{s+T(s)}^{s+R(s)} w(t) e^{-(r+\mu)(t-s)} dt \\ &= \frac{1}{r+\mu-\gamma} w(s) \left(e^{-(r+\mu-\gamma)T(s)} - e^{-(r+\mu-\gamma)R(s)} \right) \end{aligned} \tag{23}$$

Inserting (22) and (23) into (20), we get

$$R(s) = \frac{(r+\mu-\gamma)e^{-(g-\gamma)R(s)}}{\epsilon(\mu+\rho)\left[e^{-(r+\mu-\gamma)T(s)} - e^{-(r+\mu-\gamma)R(s)}\right]} \tag{24}$$

Here, the initial wage rate cancels out, such that (24) will hold for T and R independent of time of birth. This implies that agents choose the same length of education and the same retirement age, independent of when they are born. This also implies that all agents in the workforce have the same human capital and thereby the same productivity, such that individual human capital equals average human capital. Denoting these time-independent choices by T and R respectively, (24) and (19) can be written as

$$T = \frac{\alpha}{r+\mu-\gamma}\left(1 - e^{-(r+\mu-\gamma)(R-T)}\right) \tag{25}$$

$$R = \frac{(r+\mu-\gamma)e^{-(g-\gamma)R}}{\epsilon(\mu+\rho)\left[e^{-(r+\mu-\gamma)T} - e^{-(r+\mu-\gamma)R}\right]} \tag{26}$$

jointly determining the equilibrium values for education and retirement, denoted by T* and R* in the following. Here, it is assumed that $g > \gamma$, which is a standard assumption in this class of models.

2.4.1 Equilibrium

In order to examine the equilibrium in more detail, it is convenient to conduct a graphical analysis. As a first step, it is then useful to rewrite (25) and (26) such that one explicit equation for T and one explicit equation for R are obtained. These are given by

$$R = \frac{1}{r+\mu-\gamma} \cdot \left[ln\left(\frac{\alpha}{r+\mu-\gamma}\right) - ln\left(\frac{\alpha}{r+\mu-\gamma} - T\right)\right] + T \tag{27}$$

$$T = -\frac{1}{r+\mu-\gamma} \cdot ln\left(\frac{r+\mu-\gamma}{\epsilon(\mu+\rho)R} \cdot e^{-(g-\gamma)R} + e^{-(r+\mu-\gamma)R}\right) \tag{28}$$

Equation (27) will be denoted the *conditional education equation* in the following, as it was derived from the first-order condition for education, (25). It gives the optimal length of education, conditional on an optimal retirement age. Likewise, equation (28) will be denoted as the *conditional retirement equation* as it was derived from the first-order condition for retirement, (26), and it represents the optimal decision for the retirement age, given an optimal length of education.

Lemma 1. *The conditional education equation, (27), leads to a strictly convex curve giving R as a convex function of T in (T,R)-space. This curve goes through the origin and asymptotically approaches the value* $\tilde{T} \equiv \frac{\alpha}{r+\mu-\gamma}$. *The slope is larger than one at all points.*

Proof. See Appendix A. □

According to Lemma 1, $T^* < \tilde{T}$ will always hold. This is interesting, as $\tilde{T}$ is the optimal education length in the case of no retirement decision. This implies that, given the possibility to retire, agents always choose a shorter period of education.

Lemma 2. *The conditional retirement equation, (28), leads to a curve which is strictly convex, but only slightly so, in T in (T,R)-space. The curve has slope* $\lesseqgtr 1$ *as* $R \lesseqgtr \frac{1}{\mu+\rho}$ *and a positive intercept.*

Proof. See Appendix B. □

Using Lemma 1 and 2, Proposition 1, characterizing the equilibrium, can be established. Denote by c(s,t)* the optimal time path of consumption, chosen by the individual.

Proposition 1. *There exists a unique equilibrium x*=(c(s,t)*,T*,R*), where* $0 < T^* < R^*$.

Proof.

- The equilibrium path of consumption, c(s,t)*, is determined by (21) and (22).

- T* and R* are determined at the intersection of the conditional education curve and the conditional retirement curve. That there can be only one such intersection follows from Lemma 1 and 2.
- $0 < T^* < R^*$ holds. The first inequality is true as the conditional retirement curve was shown to have a positive intercept. The latter inequality is true as the conditional education curve has slope > 1.

□

The unique equilibrium, x*, is illustrated in Figure 2, below.

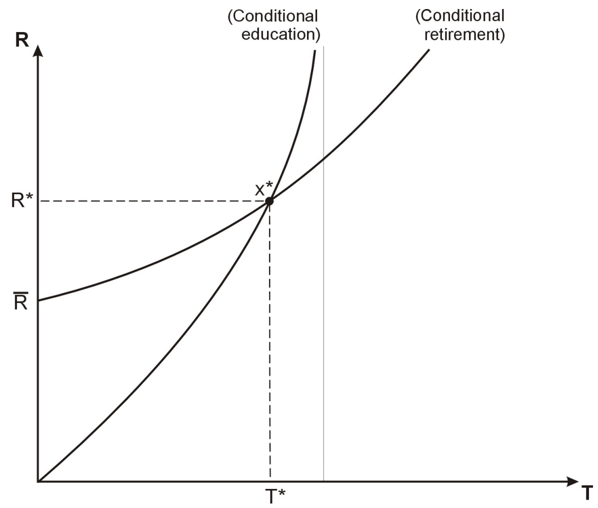


Figure 2: Determination of T* and R*

It should be noted that, although both the conditional education curve as well as the conditional retirement curve can have a slope larger than one in (T,R)-space, they will nevertheless cross, and will do so only once, as the conditional education curve asymptotically approaches the value $\tilde{T}$, whereas the conditional retirement curve is defined over the whole range of T.

The next section discusses how the equilibrium choices T* and R* are affected by changes in life expectancy, both in the sense of increased longevity and an increase in the productive horizon.

3 Analytical Results

This section shows how changes in life expectancy affect the educational as well as the retirement choice. Here, I distinguish between a reduction in mortality and a reduction in morbidity, as in Heijdra and Romp (2009b).[4] A reduction in mortality corresponds to a decrease in the parameter μ. This implies an increase in life expectancy $\frac{1}{\mu}$. As agents are in effect born as adults in the model, here a reduction in the parameter μ can be seen as a reduction in adult mortality and not in child mortality, which is the intended effect in the analysis. Thus, a reduction in mortality models an increase in longevity. In the following this will be denoted as *longevity improvements*.

The second aspect of life expectancy concerns the length of the productive life-span or the physical shape one is in. Here, this can be seen to be determined by the disutility of working. An increase in the span of productive life is thus modeled as a decrease in the disutility of working, ϵ. A decrease in this parameter implies that agents do not find it that tiresome to work. Interpreting this as an improvement in productivity throughout the entire lifetime, a decrease in ϵ can be seen as a reduction in morbidity, meaning an improvement in health conditions. Therefore, a decrease in disutility is denoted as *health improvements* in the following. The graphical analysis of the previous section serves as the basis for this discussion.

3.1 Longevity Improvements

A decrease in mortality will in general cause the present value of wage income, and thereby also the present value of lifetime labor income, to increase as agents feel more secure that they will survive and earn wage income in the future. For the education decision this implies that the investment in human capital becomes more worthwhile, as the return to education increases through the higher wage rate. Note that this is not the same as the Ben-Porath mechanism as we are concerned with the uncertainty about the length of the working life here and not about the length of work life in itself. This decrease in uncertainty would induce an increase in education upon a decrease in mortality and will be termed the *uncertainty effect* in the following.[5]

The Ben-Porath mechanism itself, however, also plays a role. The actual length of the work life will also determine the return to education. Here, the effect is ambiguous. If work life increases this will further increase the return to schooling and thus raise human capital investment. If work life in fact is decreased, this would put a halt on the mechanism and might actually reduce the return to schooling. This effect, concerned with the actual length of the work life will be referred to as the *horizon effect*.

Whether work life increases or decreases then depends on the retirement decision. Here again, there is an uncertainty as well as a horizon effect on the retirement decision as mortality decreases. Firstly, the reduction in uncertainty will cause lifetime labor income to increase, as described above. This means that agents can afford to retire earlier and will thus pull the retirement age down. As noted by Kalemli-Ozcan et al. (2010), a reduction in the

[4]The terminology and interpretation used here is slightly different, but the distinction is essentially the same.

[5]This terminology is borrowed from Kalemli-Ozcan et al. (2010), where it is used in relation to the retirement decision.

uncertainty effect makes it worthwhile to save for retirement and to actually retire. With high uncertainty of surviving through retirement, it would be too risky to save it. In this case it would be optimal to work until death.

The second effect, the horizon effect on retirement, works in the opposite direction. An increase in life expectancy implies that a longer period of consumption needs to be financed. All else equal, this would induce a later retirement age in order to earn the necessary wage income for the expected increase in consumption.

Whether the schooling period and the retirement age increase or decrease will then depend on whether the uncertainty effects or the horizon effects dominate. Based on the graphical analysis in Figure 2, the effects of a reduction in mortality are as follows.

Lemma 3. *The slope of the curve given by the conditional education equation, (27), decreases at all points upon a decrease in mortality.*

Proof. From (27)

$$\frac{\partial}{\partial \mu}\frac{\partial R}{\partial T} = \frac{T}{(\alpha + (r+\mu-\gamma)T)^2} \quad > 0 \tag{29}$$

holds. Additionally, the intercept was shown to be the origin and is, therefore, not affected. Thus, this curve shifts down when mortality is reduced. □

Lemma 4. *In the conditional retirement equation, (28), a decrease in mortality leads to a higher value of R^* for small T and a lower value of R^* for large T.*

Proof. From (28), the change in T, for given R, is given by

$$\begin{aligned}\frac{\partial T}{\partial \mu} = \frac{1}{(r+\mu-\gamma)^2} \cdot \Bigg[& \frac{(r+\mu-\gamma)\left[\epsilon(\mu+\rho)^2 R^2 + (r-\rho-\gamma)e^{(\mu+\rho)R}\right]}{(\mu+\rho)\left[(r+\mu-\gamma)e^{(\mu+\rho)R} + \epsilon(\mu+\rho)R\right]} \\ & + ln\left(e^{-(r+\mu-\gamma)R} + \frac{(r+\mu-\gamma)e^{-(r-\rho-\gamma)R}}{\epsilon(\mu+\rho)R}\right)\Bigg]\end{aligned} \tag{30}$$

□

This would be unambiguously positive if all terms in the above derivative were positive. The main concern is then whether

$$\begin{aligned} ln\left(e^{-(r+\mu-\gamma)R} + \frac{(r+\mu-\gamma)e^{-(r-\rho-\gamma)R}}{\epsilon(\mu+\rho)R}\right) & \gtreqless 0 \\ & \Leftrightarrow \\ & k \gtreqless f(R) \end{aligned} \tag{31}$$

As described in the appendix, at the intercept $k = f(R)$ will hold, whereas $k < f(R)$ will be true for larger values of R. Thus, the last term in $\frac{\partial T}{\partial \mu}$ is zero at the intercept and negative over the remaining span of R, increasing in absolute value as R increases. This implies that

$\frac{\partial T}{\partial \mu}$ will be positive for smaller values of R, close to the intercept, and is then likely to become negative for larger values of R.[6] Thus, the conditional retirement equation will change as in Figures 3 and 4, below. This leads to the following proposition.

Proposition 2. *Upon a decrease in mortality two possible cases can arise.*

- *Case 1: T^* and R^* increase*
- *Case 2: T^* increases and R^* decreases*

Proof. Follows from Lemma 3 and 4.[7] □

As longevity increases, it is thus possible that both the schooling period and the retirement age increase (Case 1), but also that schooling is prolonged in combination with earlier retirement (Case 2). This is illustrated in Figures 3 and 4 below.

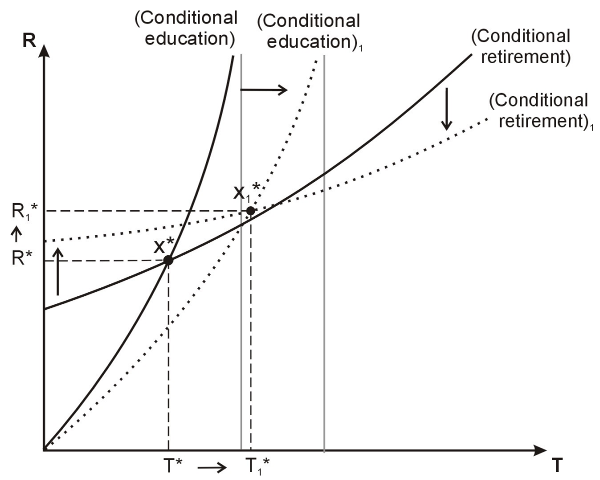


Figure 3: Case 1, effect on T* and R*, decrease in μ

[6]This is confirmed by the fact that the intercept of the conditional retirement equation increases upon a decrease in mortality. See Appendix C for details.

[7]The case of shorter education and early retirement is ruled out due to the fact that the intercept of the conditional retirement equation always increases upon an increase in life expectancy as well as the fact that the slope of the curve is positive at all points.

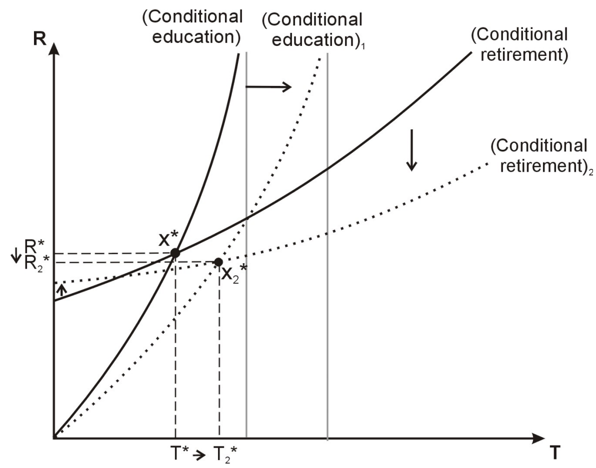


Figure 4: Case 2, effect on T* and R*, decrease in μ

In line with the discussion above, the positive horizon effect on the retirement age dominates the negative uncertainty effect in Case 1, whereas in Case 2 the opposite is true. Here, the uncertainty effect dominates and the retirement age is, thus, reduced. Concerning the education decision, the horizon effect is of ambiguous sign in Case 1. It is not clear whether the increase in the retirement age is proportional or more than proportional to the increase in life expectancy. Educational investment increases, thus either both the uncertainty and the horizon effect are of positive sign or the uncertainty effect dominates. In Case 2 the horizon effect is definitely negative, as the retirement age is reduced. As schooling nevertheless increases, the uncertainty effect must dominate.

Which of the two cases is more likely to arise depends on parameter values, as stated in Proposition 3.

Proposition 3. *Case 2, of earlier retirement, is more likely to arise when*

- *the interest rate, r, is low*
- *the rate of technological progress, γ, is high*
- *the rate of time preference, ρ, is high*

with ρ being the most important determinant for whether Case 2 occurs.

Proof. From Figure 4 it is clear the Case 2 is more likely to arise when the change in the intercept of the conditional retirement equation is relatively small. From the discussion of the intercept of the conditional retirement equation,[8] this will be the case when both the change in k and the change in f(R) are small. As $\frac{\partial k}{\partial \mu} < 0$ and $\frac{\partial f(R)}{\partial \mu} > 0$, this is true when $\frac{\partial}{\partial z}\frac{\partial k}{\partial \mu} > 0$ and $\frac{\partial}{\partial z}\frac{\partial f(R)}{\partial \mu} < 0$, where $z = (r, \gamma, \rho)$. Calculating these derivatives it is found that

[8]See Appendix C.

- $\frac{\partial}{\partial r}\frac{\partial k}{\partial \mu} = -1 \ < 0$ and $\frac{\partial}{\partial r}\frac{\partial f(R)}{\partial \mu} = 0$
- $\frac{\partial}{\partial \gamma}\frac{\partial k}{\partial \mu} = 1 \ > 0$ and $\frac{\partial}{\partial \gamma}\frac{\partial f(R)}{\partial \mu} = 0$
- $\frac{\partial}{\partial \rho}\frac{\partial k}{\partial \mu} = \frac{2r+\mu-\rho-2\gamma}{\epsilon(\mu+\rho)} \ > 0$ [9] and $\frac{\partial}{\partial \rho}\frac{\partial f(R)}{\partial \mu} = -R^3 e^{-(\mu+\rho)R} \ < 0$

such that a relatively low r, high γ or high ρ would lead to a small increase in the intercept and thereby to Case 2. However, it should also be noted that the effect of the interest rate and of γ are not expected to be that strong, as these parameters do not have an effect on the change in f(R). The main driver for early retirement will therefore be the degree of impatience, as measured by ρ. □

These results make intuitive sense as the relevant parameters affect the relative weight of the uncertainty and the horizon effect.

A lower interest rate implies that the lifetime income is discounted less heavily. This means that the present value of lifetime income is rather high. A reduction in the mortality rate, which lightens discounting even further, will then have a relatively strong effect. Additionally, the higher present value of life income implies that the horizon effect is relatively weakened. This stems from the fact that the longer period of consumption is more easily financed with a higher income. Thus, in the presence of a low interest rate, it is easier to afford a longer period of retirement and to pay for the extra periods of consumption. An increase in life expectancy is, then, likely to lead to a reduction in the retirement age as the relatively stronger uncertainty effect dominates the relatively weaker horizon effect.

The same reasoning can be used for understanding the fact that a relatively high rate of technological progress would lead to a decreasing retirement age as life expectancy increases. Note that γ is also the growth rate of the wage earned. Thus with a higher γ, the wage profile will be more upward-sloping. This implies a relatively high wage at all ages. Then the lighter discounting due to a reduction in the mortality rate, the uncertainty effect, will weigh relatively more in the retirement decision. Also again, a high γ will weaken the horizon effect, just as the lower interest rate did, due to the fact that a longer period of consumption can more readily be financed with the higher wage income. With a weaker horizon effect, and a stronger uncertainty effect the latter is more likely to dominate and earlier retirement becomes a possibility as life expectancy increases. However, as noted above, the effects from r and γ are not expected to be very strong.

The most important factor leading to earlier retirement as mortality decreases is the rate of time preference. When ρ is high, utility from consumption is discounted more heavily. Agents are relatively impatient and do not like to postpone consumption. This weakens the horizon effect as consumption in the (now longer) future does not weigh that heavily. The increase in lifetime labor income, induced by the uncertainty effect, will then be used for consumption earlier in life and for earlier retirement. Consumption later in life is relatively less important to the agent and can therefore be lower. Thus, the agent is less willing to

[9] Note that it was assumed earlier that $r + \mu - \gamma > 0$ and $r - \rho - \gamma > 0$. Together this implies that $2r + \mu - \rho - 2\gamma > 0$.

keep on working to finance consumption later in life and is more likely to retire earlier. This explains why Case 2 is more likely to arise when ρ is relatively high.

In principle also the disutility of working will be discounted more heavily, which could be thought to induce a later retirement age. An increase in life expectancy, however, does not have an effect on the relative weight of this disutility of working and does therefore not play a role in the determination of Case 1 or 2.

As noted earlier, education increases in both Case 1 and Case 2. Thus, a negative horizon effect, arising through a reduction in the working life, cannot outweigh the increase in the return to education due to reduced risk of not surviving. Although the Ben-Porath mechanism is not at work in Case 2, human capital investment increases. This shows the importance of taking uncertain lifetime into account. Whereas an increase in the average years lived might not explain the increase in educational attainment that has been seen over the last century, a reduction in the risk of not surviving the recovery period of human capital investment might be able to do so. It is also interesting to see that a reduction in the retirement age could be produced without including a public pension system or any other taxes. Merely the income effect can induce agents to choose a lower retirement age. This is an important consideration in the current public debate on retirement and increasing life-spans.

3.2 Health Improvements

In contrast to a reduction in the mortality rate which concerned the length of the actual or physical life-span, a reduction in morbidity concerns the productivity during the entire life-cycle. It should be interpreted as a general improvement in health condition, and thus an increase in the *productive* time horizon, rather than an increase in the time horizon per se. In the model this corresponds to a reduction in the parameter ϵ.

The same approach as in the previous section can be used to analyze the effects of health improvements on the education and the retirement choice. This leads to Lemma 5 and 6.

Lemma 5. *The conditional education decision is not affected by an increase in health, i.e. a decrease in* ϵ.

Proof. Follows directly from (27). □

Intuitively, this stems from the fact that the disutility of work is linearly increasing in age, beginning from the time of birth. Thus, it was assumed that one derives the same disutility from studying as from working. The reasoning was that it becomes more tiresome to work as one gets older, i.e. it is the age itself that matters for disutility not the type of activity. Everything else equal, a decrease in the disutility parameter does then not have an influence on whether the agent chooses to study or to work.

A different situation arises in the case of retirement. Here, the choice of retirement age has a direct effect on disutility, as agents do not work at all after retiring and, therefore, do not derive any disutility after leaving the workforce. The degree of disutility is, thus, of direct importance in the conditional retirement decision.

Lemma 6. *The conditional retirement curve shifts in upon an increase in health, i.e. a decrease in ϵ, in (T,R)-space.*

Proof. From (28) it follows that

$$\frac{\partial T^*}{\partial \epsilon} = \frac{1}{\epsilon \cdot \left[r + \mu - \gamma + \epsilon(\mu + \rho)R \cdot e^{-(\mu+\rho)}\right]} > 0 \tag{32}$$

Thus, T* will be smaller for a given R when ϵ is decreased. □

Proposition 4 can now be established.

Proposition 4. *Improvements in health conditions lead to an increase in the retirement age as well as an increase in the schooling period.*

Proof. Follows from Lemma 5 and 6. □

The above derived results of health improvements are illustrated in Figure 5.

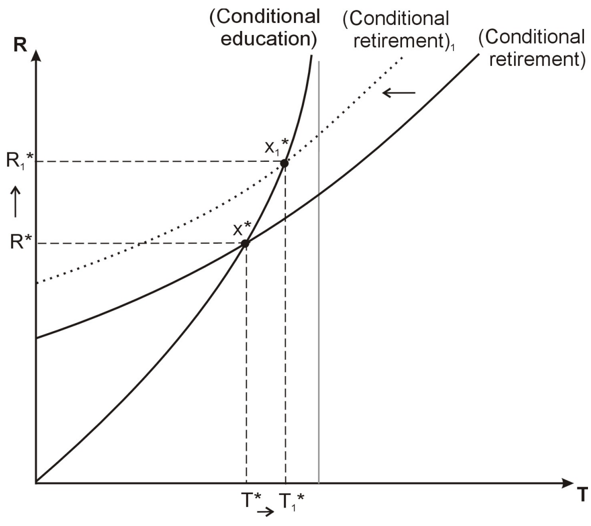


Figure 5: Determination of T* and R*, decrease in ϵ

These results are very intuitive. Improvements in health cause a reduction in the disutility of work which implies that the agent finds it reasonable to work and earn wage income for a longer time. Thus, a later retirement age will be chosen, as indicated by R_1^*. This result is essentially the same as in Heijdra and Romp (2009b). By also taking education into account, however, it is possible to see that the increase in the retirement age will also induce an increase in the schooling period, as indicated by T_1^*.

This is important to note for two reasons. On the one hand, it might entail that the length of the working life does not actually increase although the agent chooses to work longer. This will be the case if the postponement of retirement is entirely or possibly even more than offset by a longer educational period. On the other hand, health improvements will exhibit a reinforcing effect on productivity. Interpreting a reduction in disutility of working as an improvement in general health, this will in itself, and by definition, increase the productivity of the workforce. As agents will be able to work longer, it becomes more profitable to invest into education. This, in turn, will further enhance the productivity of the labor force as longer education will increase human capital in the economy. Thereby, the exogenous increase in productivity will be reinforcing by an endogenous inducement of higher human capital investment.

4 Numerical Results

In the theoretical model agents are born as adults. In the numerical simulations below, this is interpreted as an initial age of 16. This seems a reasonable age where agents are able to take their own decisions on educational choice. This goes in hand with the fact that compulsory schooling ends around this age in most western countries. Life expectancies reported in the figures are equal to $\frac{1}{\mu}$ and should thus be interpreted as life expectancy as seen from age 16. The chosen parameter values for the benchmark case are reported in Table 1 below. In general, the simulation results should be seen as pertaining to different countries with different sets of parameter values, as they are not concerned with transitionary dynamics.

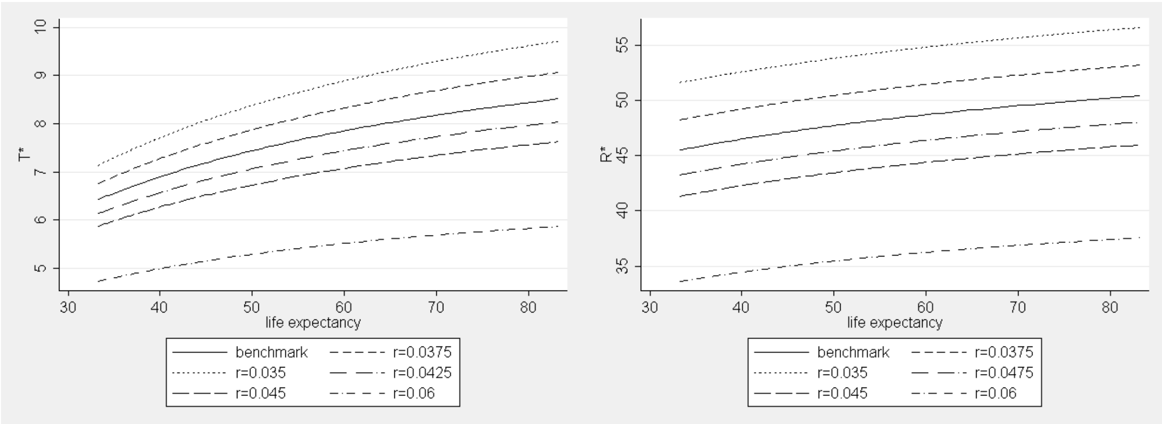
r	0.04
γ	0.015
ρ	0.02
ε	0.031
α	0.4

Table 1: Parameter values, benchmark case

The value of the interest rate is based on Heijdra and Romp (2008). The rate of technological progress is chosen to be slightly higher than in Gertler (1999), in accordance with OECD estimates of labor productivity growth over the past ten years (OECD.Stat).[10] The rate of time preference is set to a value lying between Heijdra and Romp (2008) and Heijdra and Romp (2009b), where ρ is set to 0.0159 and 0.039 respectively. The value of ϵ is calibrated such that the retirement age equals 65 at a life expectancy of 80 years in the benchmark case. The parameter value for α is based on estimates in Ciccone and Hall (1996).

[10]See Appendix D for data sources.

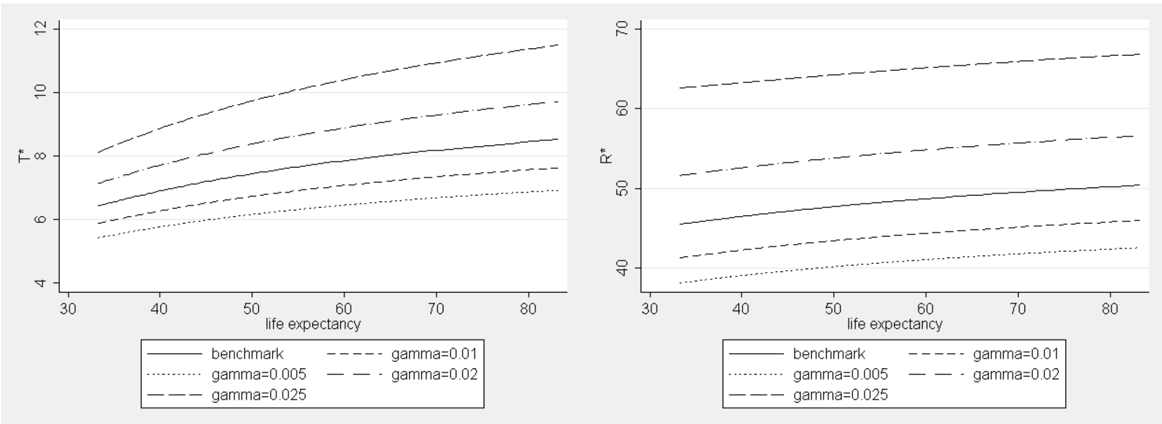
The following figures show how the optimal values for educational length and the retirement age vary with varying parameter values. The benchmark case is included in every figure as a reference.



(a) effect on T*

(b) effect on R*

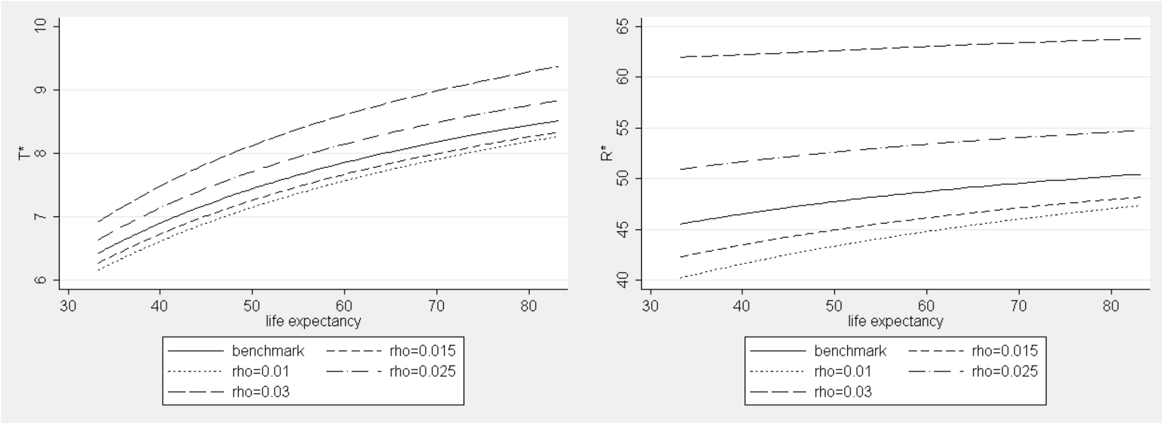
Figure 6: Change in r



(a) effect on T*

(b) effect on R*

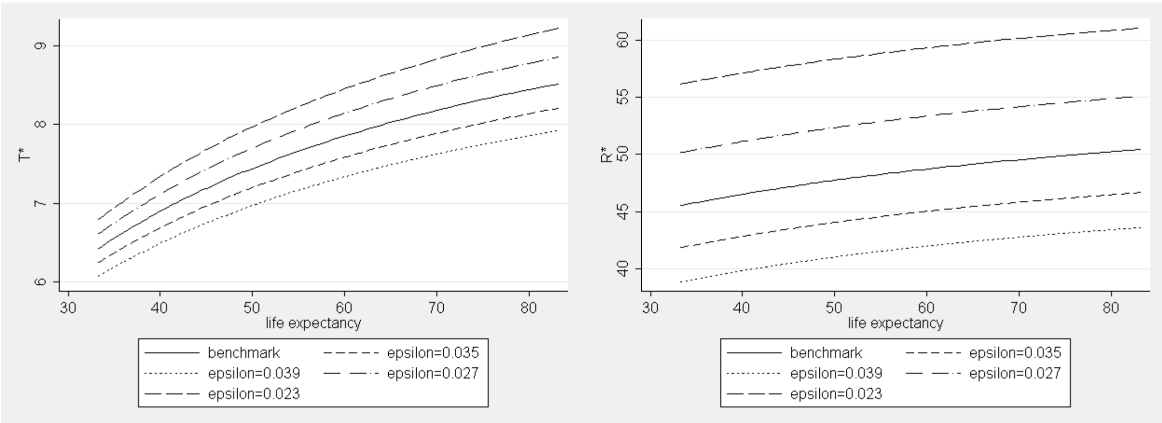
Figure 7: Change in γ



(a) effect on T*

(b) effect on R*

Figure 8: Change in ρ



(a) effect on T*

(b) effect on R*

Figure 9: Change in ϵ

Figures 6 and 7 confirm that the effects of the interest rate and of the technological growth rate on early retirement are not that strong. This can be seen from the fact that the slope of R* does not seem to change when r is relatively low or when γ is relatively high. The results also show that a higher interest rate in general will lead to a lower retirement age. This can be explained by the fact that a higher r implies a higher growth rate of consumption. Thus, the individual consumption profile is rather upward sloping and consumption will be quite high at older ages. This in turn means that the marginal utility derived from an extra unit of consumption is relatively low. This induces agents to stop working earlier, as the utility derived from working longer is rather low.

Also a relatively low rate of technological progress can be seen to induce a lower level of the retirement age. A low level of γ implies a relatively low wage at older ages. Thus, working does not pay off that much and an earlier retirement age is chosen.

Figure 8 shows that a higher ρ is more likely to lead to earlier retirement, as described in Section 4.1. However, also for a rate of time preference of 3%, the slope is still positive. Thus, Case 2 does not seem too likely to arise. Moreover, it can be seen that a higher ρ will in general imply a higher retirement age as a higher ρ implies a lower growth rate of consumption. The reasoning here is analogue to the one for the interest rate. Given a rather low growth rate of consumption, the marginal utility of an extra unit of consumption will be relatively high. This induces the agent to continue working, i.e. to choose a higher retirement age. The fact that a higher ρ also means that disutility from working is discounted more heavily works in the same direction, as agents do not find it that tiresome to work at older ages and therefore postpone retirement. The higher retirement age then imposes a general equilibrium effect on the schooling decision, as the rate of time preference in itself does not effect the length of schooling. Later retirement prolonges the recovery horizon of human capital investment and, thus, leads to longer schooling as evident from figure 8.

The results for a change in ϵ, as shown in figure 9, are also as expected. A lower ϵ induces both a longer schooling period and later retirement. In addition, both profiles can be seen to be upward sloping as life expectancy increases.

The above also shows that the distinction between length of life and length of working life is not that important in early stages of development, i.e. at low values of life expectancy. Here, agents choose to work until they die, in that $R^* > \frac{1}{\mu}$. Soon, however, it is optimal to retire, highlighting the importance of the correct definition of the recovery horizon of human capital investment when using the Ben-Porath mechanism.

In general, the numerical examples above indicate a rather strong positive effect on schooling. If the mortality rate was increasing in age and mortality reductions would take place especially at older ages, this effect would be smaller. This is due to the fact that the uncertainty effect would be weaker in this case.

The effect on the length of work life in these numerical examples is summarized in figure 10, below. Here the benchmark case is included, as well as the two most 'extreme' scenarios from the parameter changes above.

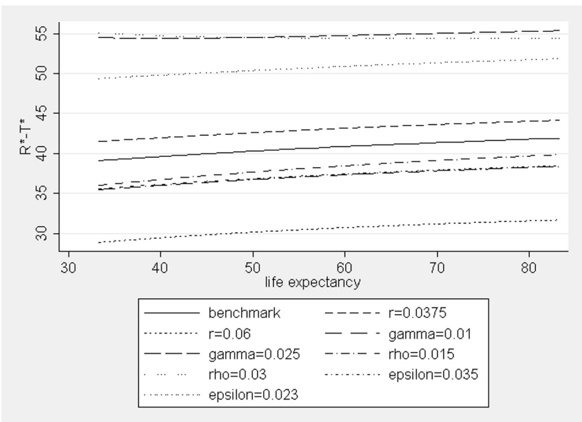


Figure 10: Effect on length of worklife

It appears that in general the effect on the length of work life is, in fact, positive. This is opposed to previous work, e.g. Boucekkine et al. (2002), who find that the retirement age increases proportionally with schooling. With a positive effect on the length of work life, however, the growth effect of an increase in life expectancy would be positive as the workforce would both be more productive and larger. In the case of a high rate of time preference, ρ, however the effect on the length of work life is actually negative. Thus, although both schooling and the retirement age increased in this case, the retirement age increased less than proportionally. In this case the growth effect is potentially negative, as the increase in the productivity of the workforce might be more than offset by a smaller size of the workforce.

5 Conclusion

Examining the Ben-Porath mechanism when retirement is taken into account has shown three major points.

Firstly, an increase in life expectancy, as reflected by a decrease in mortality, does not necessarily imply an increase in work life. Thus, the recovery horizon of human capital investment does not necessarily increase upon an increase in life expectancy. Using the Ben-Porath mechanism implicitly assuming this may then lead to wrong conclusions. Not only is it not clear whether the retirement age increases proportionally with an increase in schooling, but even early retirement can arise under certain circumstances, especially when agents are very impatient. However, numerical examples showed that the most likely scenario is that of postponed retirement when life expectancy increases. Nevertheless, also earlier retirement is a possibility and retirement should thus not be neglected. It is not clear whether the increase in the productivity of the workforce can outweigh the reduction in the size when early retirement arises and it is not given that retirement is postponed one-for-one with the increase in life expectancy. This remedies the unambiguously positive growth effect found when the Ben-Porath mechanism is analyzed without retirement. it is also interesting to note that a falling retirement age could be produced without including a public pension system or any kind of taxes. If the income effect is strong enough, a lower retirement age can be induced.

Secondly, it was shown that educational attainment nevertheless increases upon a decrease in mortality. This is however not due to the increase in the actual length of the investment recovery period but due to a reduction in survival uncertainty. Thus, even if the Ben-Porath mechanism is not at work, reduced risk of mortality induces higher investment in human capital. As long as the *expected* length of work life increases, will also schooling increase. It is therefore of importance to take the uncertainty about life-spans into account.

Thirdly, it was found that a reduction in morbidity would have a reinforcing effect on the productivity of the workforce. Not only would a healthier workforce be more productive in itself, this also makes it more worthwhile to invest in schooling, thereby increasing productivity even further.

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References

[1] Acemoglu, D. and Johnson, S., 2007. 'Disease and Development: The Effect of Life Expectancy on Economic Growth'. *Journal of Political Economy*, Vol. 115, No. 6, 925-985.

[2] Barro, R. J. and Lee, J.-W., 2000. 'International Data on Educational Attainment: Updates and Implications'. CID Working Paper No. 42.

[3] Ben-Porath, Y., 1967. 'The Production of Human Capital and the Life Cycle of Earnings'. *The Journal of Political Economy*, Vol. 75, No. 4, Part 1, 352-365.

[4] Blanchard, O. J., 1985. 'Debt, Deficits, and Finite Horizons'. *Journal of Political Economy*, vol. 93, no.2.

[5] Bloom, D. E., Canning, D., Mansfield, R. K., Moore, M., 2007. 'Demographic change, social security systems and savings'. *Journal of Monetary Economics* 54, 92-114.

[6] Boucekkine, R., de la Croix, D., and Licandro, O., 2002. 'Vintage Human Capital, Demographic Trends, and Endogenous Growth'. *Journal of Economic Theory* 104, 340-375.

[7] Cervellati, M. and Sunde, U., 2009. 'Life Expectancy and Economic Growth: The Role of the Demographic Transition'. IZA Discussion Paper No. 4160.

[8] Cervellati, M. and Sunde, U., 2010. 'Longevity and Lifetime Labor Supply: Evidence and Implications revisited'. Working Paper.

[9] Ciccone, A. and Hall, R.E., 1996. 'Productivity and the Density of Economic Activity'. *The American Economic Review*, Vol. 86, No. 1, pp. 54-70.

[10] Echevarria, C. A., 2004. 'Life expectancy, schooling time, retirement, and growth'. *Economic Inquiry*, Vol. 42, No. 4, 602-617.

[11] Gertler, M., 1999. 'Government debt and social security in a life-cycle economy'. Carnegie-Rochester Conference Series on Public Policy 50, 61-110.

[12] Hansen, C.W. and Lønstrup, L., 2012. 'Can higher life expectancy induce more schooling and earlier retirement?'. *Journal of Population Economics*, 25(4), 1249-1264.

[13] Hazan, M., 2009. 'Longevity and Lifetime Labor Supply'. *Econometrica*, Vol. 77, No. 6, 1829-1863.

[14] Heijdra, B. and Romp, W. E., 2009a. 'Human capital formation and macroeconomic performance in an ageing small open economy'. *Journal of Economic Dynamics & Control* 33, 725-744.

[15] Heijdra, B. and Romp, W. E., 2009b. 'Retirement, pensions, and ageing'. *Journal of Public Economics* 93, 586-604.

[16] Hurd, M., 2008. 'Retirement'. *The New Palgrave Dictionary of Economics*, Second Edition.

[17] Jayachandran, S. and Lleras-Muney, A., 2009. 'Life Expectancy and Human Capital Investments: Evidence from Maternal Mortality Declines'. *The Quarterly Journal of Economics*, 124(1), 349-397.

[18] Kalemli-Ozcan, S., Ryder, H. E., and Weil, D. N., 2000. 'Mortality decline, human capital investment, and economic growth'. *Journal of Development Economics*, Vol. 62, 1-23.

[19] Kalemli-Ozcan, S., and Weil, D. N., 2010. 'Mortality change, the uncertainty effect, and retirement'. *Journal of Economic Growth*, Vol. 15, No. 1, 65-91.

[20] Oeppen, J. and Vaupel, J. W., 2002. 'Broken Limits to Life Expectancy'. *Science*, Vol. 296, 1029-1031.

[21] Romer, D., 2006. '*Advanced Macroeconomics*'. New York, McGraw-Hill, 3rd Edition.

[22] Sheshinski, E., 2009. 'Uncertain Longevity and Investment in Education', CESifo Working Paper No. 2784.

[23] Sheshinski, E., 2007. 'Longevity and Aggregate Savings', Working Paper.

[24] Strulik, H. and Werner, K., 2012. 'Life expectany, labor supply, and long-run growth: reconciling theory and evidence', cege discussion paper, Number 141.

[25] Yaari, M. E., 1965. 'Uncertain life time, life insurance and the theory of the consumer'. *Review of Economic Studies* 32, 137-150.

Appendix

A Proof of Lemma 1

- $T < \frac{\alpha}{r+\mu-\gamma}$ will always hold as $ln\left(\frac{\alpha}{r+\mu-\gamma} - T\right)$ in (27) would otherwise not be defined
- That the slope of (27) is positive and larger than one in (T,R)-space can be seen from

$$\frac{\partial R}{\partial T} = 1 + \frac{1}{(r+\mu-\gamma)(\frac{\alpha}{r+\mu-\gamma} - T)} \quad > 0, \ > 1 \tag{A.1}$$

- (27) is a convex function in T, as seen from differentiating (A.1) once more w.r.t. T

$$\frac{\partial^2 R}{(\partial T)^2} = \frac{1}{(r+\mu-\gamma)\left(\frac{\alpha}{(r+\mu-\gamma)} - T\right)^2} \quad > 0 \tag{A.2}$$

- (27) goes through the origin. Setting T to zero in the conditional education equation gives R=0.

B Proof of Lemma 2

- First it should be noted that

$$\begin{gathered} \frac{r+\mu-\gamma}{\epsilon(\mu+\rho)} e^{-(g-\gamma)R} + e^{-(r+\mu-\gamma)R} \ \leq \ 1 \\ \Leftrightarrow \\ \underbrace{\frac{r+\mu-\gamma}{\epsilon(\mu+\rho)R}}_{\equiv k} \ \leq \ \underbrace{(1-e^{-(r+\mu-\gamma)R})Re^{(g-\gamma)R}}_{\equiv f(R)} \end{gathered} \tag{B.1}$$

  is required to ensure a positive value of T. That this indeed will be the case, remains to be shown.

- The slope of (28) is positive in (R,T)-space and thereby also positive in (T,R)-space. This can be seen from the fact that differentiating (28) with respect to T gives

$$\frac{\partial T}{\partial R} = \frac{\epsilon(\mu+\rho)R^2 + e^{(\mu+\rho)R}(1+(g-\gamma)R)}{\epsilon(\mu+\rho)R^2 + e^{(\mu+\rho)R}(r+\mu-\gamma)R} \quad > 0 \tag{B.2}$$

  which is positive. In addition, it can be noted that this slope will be > 1 in (R,T)-space (and thus < 1 in (T,R)-space) for $R < \frac{1}{\mu+\rho}$. For a value of $R > \frac{1}{\mu+\rho}$ the slope will be < 1 in (R,T)-space (i.e. > 1 in (T,R)-space).

- The above already indicates that the conditional retirement equation is a concave function in (R,T)-space. This is confirmed by the second derivative

$$\frac{\partial^2 T}{(\partial R)^2} = \\ -\frac{e^{2(\mu+\rho)R}(r+\mu-\gamma)+\epsilon e^{(\mu+\rho)R}(\mu+\rho)\left[2-2(\mu+\rho)R+(\mu+\rho)^2R^2\right]}{R^2\left[e^{(\mu+\rho)R}(r+\mu-\gamma)+\epsilon(\mu+\rho)R\right]^2} \tag{B.3}$$

being negative for $R > \frac{1}{\mu+\rho}$ as well as $R > \frac{2}{\mu+\rho}$. In between these values the function might be convex, however this would not matter for the results. Additionally, the derivative was found to be negative and close to zero by numerical examples. Slight concavity in (R,T)-space implies slight convexity in (T,R)-space.

- The conditional retirement decision has a positive intercept in (T,R)-space. This can be seen from the fact that T=0 can only be a solution to (28) when

$$\frac{r+\mu-\gamma}{\epsilon(\mu+\rho)} = (1-e^{-(r+\mu-\gamma)R})Re^{(g-\gamma)R} \\ \Leftrightarrow \\ k = f(R) \tag{B.4}$$

f(R) will equal the positive value k for a positive value of R, since f(R) is an increasing function in R, as seen from

$$\frac{\partial f(R)}{\partial R} = (r+\mu-\gamma)R\cdot e^{-(\mu+\rho)R}+(1-e^{-(r+\mu-\gamma)R})(1+(r-\rho-\gamma)R)e^{(r-\rho-\gamma)R} \quad > 0 \tag{B.5}$$

Furthermore, the second derivative of f(R) w.r.t. R is given by

$$\frac{\partial^2 f(R)}{(\partial R)^2} = e^{-(\mu+\rho)R}(2(\mu+\rho)-(\mu+\rho)^2R)+(r-\rho-\gamma)(2+(r-\rho-\gamma)R) \tag{B.6}$$

which is $\gtreqless 0$ for $R \lesseqgtr \frac{2}{\mu+\rho}$. Thus, f(R) will be convex for $R < \frac{2}{\mu+\rho}$ and concave for $R > \frac{2}{\mu+\rho}$. Hence, the shape of f(R) in Figure B.1, which illustrates the determination of the intercept.

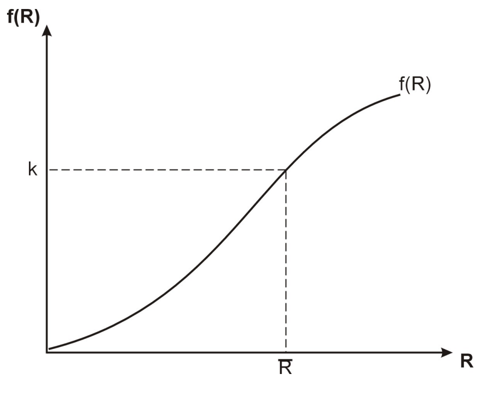

Figure B.1: determination of the intercept, $\bar{R}$

The intercept, denoted $\bar{R}$ in the following, is determined where f(R)=k. This shows that the intercept of the conditional retirement equation will always be positive.

- The previous point also shows that the condition for a non-negative value of education mentioned above, $k \leq f(R)$ will always hold. At the intercept we have $k = f(R)$. Then, since f(R) is an increasing function in R and k is constant with respect to R, $k \leq f(R)$ is fulfilled over the remaining span of the conditional retirement equation.

C Change in the intercept of (28)

As described earlier, the intercept of the conditional retirement equation is determined by the function f(R) and the value k. Both of these are dependent on the mortality rate, μ, according to

$$\frac{\partial f(R)}{\partial \mu} = e^{-(\mu+\rho)R}R^2 \quad > 0 \tag{C.1}$$

$$\frac{\partial k}{\partial \mu} = -\frac{\epsilon(r-\rho-\gamma)}{(\epsilon(\mu+\rho))^2} \quad < 0 \tag{C.2}$$

This implies that f(R) shifts down and k increases upon a decrease in μ. Thereby, the intercept increases from $\bar{R}$ to $\bar{R}_1$, as illustrated in Figure C.1 below. Here, μ is decreased by one unit in both $f_1(R)$ and k_1.

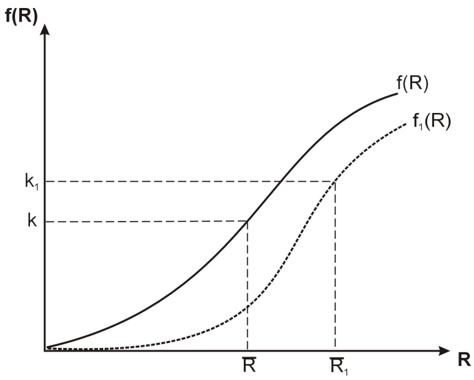

Figure C.1: change in intercept of conditional retirement, decrease in μ

D Data Appendix

Labour productivity growth: 'Labour productivity annual growth rate'. OECD.Stat, productivity, labour productivity growth.